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*I am greatly indebted to Martin Feldstein and Joel Slemrod for helpful discussions. I would also like to thank Y. Ben Porath, M. Bruno, H. J. Buchner, and R. Musgrave who gave helpful comments on earlier drafts.

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Forthcoming: Public Finance Quarterly
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ABSTRACT

A simple model of tax evasion is presented leading to the following conclusions:

1. Tax evasion is a function of *inter alia* the marginal tax rate only if the probability of being caught is a function of undeclared income.

2. Tax evasion introduces uncertainty which causes a utility loss. An example is presented showing that the excess burdens of tax evasion and the income tax are sometimes additive. Thus ignoring the excess burden of tax evasion will usually lead to underestimation of the total excess burden of a tax.
ON THE EXCESS BURDEN OF TAX EVASION

Although it is generally agreed that tax evasion leads to welfare loss, economists have not tried to formalize this excess burden. Weiss (1976) has recently challenged the popular view, suggesting that societies may actually benefit from allowing the taxpayer to cheat.

The purpose of this paper is to formulate the excess burden of tax evasion. We first examine the causes of tax evasion in a simple model similar to that of Allingham and Sandmo (1972). The simple model assumes that true income and the probability of detection are given. The main conclusions from the simple model are: (1) The marginal tax rate does not affect the existence of tax evasion, but (2) it does affect the extent of the evasion through an income effect. Later the assumption of a given probability of detection is relaxed leaving the first conclusion unaffected.

In the second section we formulate the definition of excess burden resulting from tax evasion. In the third section we relax the assumption of a given true income, by allowing variable labor supply. The conclusions from this section are that the excess burden of tax evasion and the tax rate are sometimes additive, and that in any case the combination of the two is usually greater than the effect of the tax burden alone. This means that to use the latter understates the total excess burden. The last conclusion leads us to question Weiss' results. Although we have not succeeded in disproving Weiss' contention, we do show that the examples he cited fail to substantiate it.
I. A SIMPLE MODEL OF TAX EVASION

Assume as in the Allingham and Sandmo (1972) model (A-S) that the taxpayer chooses his declared income so as to maximize

\[
\text{Max } E(U) = (1-P)U[y-T(x)] - PU(y-T(x) - F[T(y)-T(x)])
\]

where \( x \) is declared income

\( y \) is true income, in this section assumed to be given

\[ F = 1 + \pi, \text{ where } \pi \text{ is the penalty rate} \]

\( U \) is assumed to be a well-behaved utility function, that is \( U' > 0; U'' < 0 \)

\( T(\ ) \) is the tax function, \( T' > 0, T'' > 0 \)

\( P \) is the probability of being caught, in this section assumed to be constant.

This model differs slightly from the A-S model. Here, the penalty paid by the taxpayer is a function of the tax evaded, while in the A-S model the penalty is on the income evaded. \(^{1/} As shown later, imposing the penalty rate on the evaded tax simplifies the analysis while leaving most of the results unchanged.
For our purpose it is convenient to present the model in net income terms. Thus (1) can be rewritten as:

\[ \max_{g} E(U) = (1-P)U(c+g) + PU(c-ng), \]

where \( g = T(y) - T(x) \) is the tax evaded and \( c = y - T(y) \) is the after tax true (net) income. In the case that tax evasion is zero \( c \) is net income. The first order condition for maximum is:

\[ (1-P) U'(c+g) = PU'(c-ng) \]

The second order condition for maximum is fulfilled if \( U'' < 0 \). That is if the taxpayer is risk averse.

By differentiating (3) with respect to \( P \) and \( \pi \) it can be shown that \( \frac{\partial g}{\partial P} < 0 \) and \( \frac{\partial g}{\partial \pi} < 0 \) so that an increase in \( P \) or an increase of the penalty rate decrease tax evasion. Evaluating (3) at \( g=0 \), we obtain the condition necessary for tax evasion to occur:

\[ 1 - PF > 0 \]

Condition (4) shows that the occurrence of tax evasion is independent of the tax rate. It explains why tax evasion is widespread: even if the penalty rate is as high as 400 percent, the probability of detection would have to exceed 0.2 in order for tax
evasion to be prevented. (It is easy to think of examples such as tipping where detection is presumably less probable.)

A useful way of viewing (4) is to consider that \( 1 - PF \) constitutes the condition for a fair gamble, i.e. whose expected value is zero. Thus (4) implies that there will be tax evasion whenever the gamble is favorable to the taxpayer. Its expected value is

\[
E(g) = (1-PF)g, 
\]

where \( (1-PF) > 0 \) ensures that it is positive.

This outcome is well recognized in the literature dealing with risk and in Arrow's phrase "a risk averter takes no part of an unfavorable or barely fair gamble; on the other hand he always takes some part of a favorable gamble" (Arrow, 1970, p. 100, italics in source).

The relationship between the optimum gamble, \( g \), and income can be seen in (3). Differentiating (3) with respect to net true income \( c \), we obtain

\[
\frac{\partial g}{\partial c} = \frac{1}{D} (1-P)U'(c+g)[R_A(c-\pi g) - R_A(c+g)], 
\]

where \( D = (1-P)U''(c+g) + PfU''(c-\pi g) < 0 \) and \( R_A = -U''/U' \), the absolute risk aversion. Thus as income increases, \( \frac{\partial g}{\partial c} \geq 0 \) according
as the absolute risk, $R_A$ decreases, is constant, or increases, with income.

A second way of obtaining the sign of \( \frac{\partial g}{\partial c} \) will be useful later. The definition of nonincreasing global risk aversion is "for every pair of wealth levels, \( W \) and \( W' \) (\( c \) and \( c+\Delta c \) in our notation) satisfying \( W' > W \), and for every gamble \( x \), if \( x \) is accepted at the wealth level \( W \), then it is also accepted at the wealth level \( W' \)" (Yaari, 1969, p. 322, axiom V). Thus for a taxpayer with nonincreasing risk aversion, if the gamble \( g \) is accepted for income level \( c \), it will also be accepted for income \( c + \Delta c \).

Differentiating (3) with respect to \( \phi \), where \( \phi \) is any parameter in the tax function, we obtain

\[
\frac{\partial g}{\partial \phi} = \frac{(1-P)U''(c+g) - P\pi U''(c-\pi g)}{(1-P)U''(c+g) + P\pi^2 U''(c-\pi g)} \frac{\partial c}{\partial \phi} \frac{\partial c}{\partial \phi}.
\]

From (7) we observe that a change in the tax function which does not alter true net income does not change the amount of tax evaded. Thus we can conclude that an income compensated change in the marginal tax rate (that is a change in the marginal tax rate accompanied by a compensation necessary to keep true net income constant) does not affect the amount of tax evaded. Thus, a change in the marginal tax rate affects the amount of tax evaded through its income effect. The income evaded is the medium used by the taxpayer to choose the size of the gamble and depends on the marginal tax rate.
The assumption that the probability of being caught is independent of the amount of income evaded seems very unrealistic. Usually the tax authorities have some idea of the taxpayer's true income, and it is reasonable to assume that the probability of being caught is an increasing function of the undeclared income (or of the ratio of undeclared to true income as in Srinivasan, 1973). In this case the marginal tax rate would affect the amount of tax evaded indirectly through its income effect and directly through its correlation with the probability of detection.

Let $g_y = y-x$ be undeclared income. Then the taxpayer's problem is

\[
\text{Max } \mathbb{E}(U) = \left[1-P(g_y)\right]U(c+g) + P(g_y)U(c-ng),
\]

where $P' > 0$ and $P'' > 0$.

The first-order condition for a maximum is

\[
\frac{\partial \mathbb{E}(U)}{\partial g} = (1-P)U'(c+g) - PuU'(c-ng) - \frac{P'}{T'}[U(c+g) - U(c-ng)]=0,
\]

where $T'$ is the marginal tax rate. Evaluating (3') at $g = 0$, we again arrive at $1-P(0)F > 0$ as the condition for tax evasion. Thus, the existence of tax evasion is independent of the marginal tax rate while its size is a function of the marginal tax rate. The relation between tax evasion and the marginal tax rate can be seen from the following
exercise. Assume that \( g^* \) is the optimal tax evasion. Assume that we make a compensated increase in the marginal tax rate. That is, we increase the marginal tax rate and compensate the taxpayer so his net income \( c \) does not change. Thus for any given \( g, g_y \), the income evaded, is smaller and thus \( P \) and \( P' \) are lower. Thus from (3') we can see that \( \frac{\partial E}{\partial g} \bigg|_{g^*} > 0 \) which means that the optimal tax evaded should be increased. Thus, we can conclude that the higher the marginal tax rate, other things being equal, including net true income, the more tax is evaded. The explanation of this outcome is that the higher the marginal tax rate, the smaller the amount of undeclared income necessary to create a given gamble in terms of the tax evaded. Since the probability of detection is assumed to be positively related to the amount of undeclared income, higher marginal tax rate means a greater probability of getting away with a given gamble.

The second-order condition for a maximum is ensured by the assumptions. As for the sign of \( \partial g/\partial c \), it is complicated to compute directly but we can use the same argument as before; we also need \( \partial P/\partial c \leq 0. \)

Assume that absolute risk aversion decreases as income increases. Then if the optimum gamble at income level \( c \) is \( g \), the same gamble will be accepted at income level \( c + \Delta c \). Since \( P \) is a non-increasing function of \( c \) (because the marginal tax rate is an increasing function of income) there is an additional inducement to increase the gamble. Thus, we can conclude that \( g \) increases with income for a
taxpayer whose absolute risk aversion decreases as income increases. \(5\) (For a taxpayer whose risk aversion increases with income nothing can be said about \(\frac{3g}{3c}\) without additional assumptions).

II. THE EXCESS BURDEN OF TAX EVASION

The excess burden of a tax system is the loss in utility beyond that which would be incurred if a lump-sum tax were collected. \(6\) In the case of tax evasion the excess burden occurs because of the uncertainty introduced into the economy by tax evasion. The excess burden of tax evasion can be defined as the difference between the utility obtained under a tax system with tax evasion and that obtained under a system in which the taxpayers agree not to cheat while the government gets the same average tax (including the penalties on tax evasion). The mean income of the taxpayer and of the government would be the same under both these systems. Thus the excess burden of tax evasion would be the utility loss due to the riskiness of tax evasion. \(7\) Formally

\[
B = U[c + (1-P\pi)g] - (1-P)U(c+g) - PU(c-\pi g).
\]

Graphically we can represent the utility loss due to tax evasion by KL in Figure 1, while LM is the additional tax that could be levied in a system with no tax evasion without reducing the utility to

[Figure 1 about here.]
Figure 1. The Excess Burden of Tax Evasion
below that occurring under tax evasion. We have shown in Section I that
tax evasion produces an income effect. In Figure 1 this income effect
is reflected by the movement of \( C + (1 - PF)g \), the point on the horizontal
axis at which we measure the excess burden. A compensated change in
\( p \) (or \( \pi \)) means that we measure the excess burden of tax evasion at the
same point \( c \). Since \( \frac{\partial g}{\partial P} < 0 \) (and \( \frac{\partial g}{\partial \pi} < 0 \)) an increase in \( P \) decreases \( g \)
and thus the excess burden measured at point \( c \) declines. Thus we can
conclude that a compensated decrease in \( P \) (or in \( \pi \)) augments the excess
burden of tax evasion. This outcome lies behind the definition of the
excess burden of tax evasion.

Assume that \( P \) is associated with administrative costs. The
government can raise liabilities \( T \) by different combinations of \( P \) and
\( t \). In this model changing \( t \) will not result in any excess burden.
Raising \( P \) means raising administrative costs and thus entails a real
loss to the economy. On the other hand decreasing \( P \) raises \( g \) and thus
enlarges the excess burden of tax evasion. In an optimum solution the
marginal administrative cost of raising a dollar through an increase in
\( P \) should be equal to the marginal excess burden caused by an increase in
the tax rate enough to raise a dollar. 8/

III. TAX EVASION WITH VARIABLE LABOR SUPPLY

In a model with variable labor supply, the taxpayer has two
decision variables; the amount of tax evaded \( g \), and the amount of labor
\( h \) (or leisure, \( h = 1 - \lambda \)). The two decisions are not independent of
one another, and thus in general the excess burden of tax evasion is not independent of the excess burden of the income tax. We would like to demonstrate that the dependence between the two decisions is mainly through the income effect of the tax evasion. Thus if one neutralizes the income effect, then the excess burden of tax evasion and the excess burden of the income tax do not depend one on the other and we can simply add the two excess burdens in order to obtain the excess burden of the tax systems.

With an incentive to cheat and variable labor supply, the tax-payer's problem is

\[
\max_{g,\lambda} (1-P)U(c+g,\lambda) + PU(c-ng,\lambda)
\]

S.t. \(c = (1-t)h,\)

(9)

where \(P\) is the probability of being caught and \(g\) is the tax evaded, \(t\) is the tax rate and for simplicity's sake \(t\) and \(P\) are constants. The first-order conditions are

(10) \( (1-P)U'_1(c+g,\lambda) = Pu'_1(c-ng,\lambda) \)

(11) \((1-t)[(1-P)U'_1(c+g,\lambda)+PU'_1(c-ng,\lambda)] = (1-P)U'_2(c+g,\lambda)+PU'_2(c-ng,\lambda) \)

As can be seen from (10) \(1-PF > 0\) ensures that \(g > 0\). Condition (11) differs from the first-order condition in a model without tax evasion.
only in that it contains mean marginal instead of marginal utilities. Assume that the utility function is separable in income and leisure, then the marginal utility of leisure is independent of income. Assume also that the marginal utility of the mean income is equal to the mean marginal utility of income as in the quadratic function
\[ U(c,\ell) = \alpha + \beta c + (1/2\gamma)c^2 + \delta \ell. \]
Then the first-order conditions for maximization [according to (10)-(11)] are

\[ (12) \quad (1-P)U'_1(c+g) = P\pi U'_1(c-\pi g) \]

\[ (13) \quad (1-t)U'_1[c + (1-PF)g] = U'_2(\ell). \]

Condition (13) would also apply if the government imposed an income tax rate \( t \) and gave a lump sum of \((1-PF)g\) to the taxpayer. Thus we can compute the excess burden resulting from the uncertainty as \( B_e = U[c + (1-PF)g] - (1-P)U(c+g) - P\pi U(c-\pi g) \), and then compute the excess burden resulting from the income tax, the latter being independent of the former. The excess burden of the tax system will then be the sum of the two. In the general case where the utility function is not separable in income and leisure and not quadratic in income, we can view the summation of the excess burdens as a first-order approximation to the excess burden of the tax system. 9/

Before we proceed, let us illustrate the magnitude of the excess burden of tax evasion. Assume that the quadratic utility function is the following: \( U = 2c - 2c^2 + .5\ell \), while the wage rate is
equal 1. Assume that $t = .4; p = .7$ and $\pi = 1$. Then the maximum utility is 
0.58333. A lump-sum which raises the same revenue, yields a utility 
level of 0.63344 while eliminating evasion and raising the same revenue 
yields a utility of 0.61395. Thus the excess burden of tax evasion may 
be of the same magnitude of the excess burden of the income tax. More­
over, it may be argued that tax evasion may cause another type of excess 
burden (Alm, 1985), due to inefficient allocation of resources by tax­
payers who try to evade. Hence, we may conclude that ignoring the ex­
cess burden of tax evasion may cause underestimation of the excess bur­
den of a tax system.

Weiss (1976) argues that by creating an incentive to cheat the 
government may improve the welfare of risk-averse individuals. In 
other words, there exists a combination of $t_e, p_e, F_e$ such that $P_e F_e < 1$ 
that increases welfare beyond that achieved by a tax rate $t$ and $PF > 1$, 
and at the same time raises the same revenue. This means that the ex­
cess burden of a tax system that encourages evasion can be lower than 
that of a system which discourages it (with tax revenue held constant).

This argument, if it is valid, raises some doubts with regard to the optimality of a policy that tries, subject to a given resource 
constraint, to minimize tax evasion. Therefore it is worthwhile to re­
examine it.
The direct way to verify this argument is to solve the government's problem in an explicit model. Let \( R(t, \pi, P) \) be the expected revenue raised

\[
R(t, \pi, P) = \frac{t}{1-t} c - (1-P)g
\]

Then the problem is

\[
\max_{t, \pi, P} \left( (1-P)U(c^*+g^*, \lambda^*) + PU(c^*+\pi g^*, \lambda^*) \right)
\]

\[
\text{s.t. } R(t, \pi, P) = T_0
\]

where \( c^*, g^*, \lambda^* \) are the solutions to equations (10) and (11).

In order to simplify the problem, let us assume that \( P \) is given and that the utility function is of the form \( U(c) + \delta \lambda \). Then the first order conditions for optimum are

\[
P g^* U'(c^*+\pi g^*) = \lambda \frac{\partial R}{\partial \pi}
\]

\[
\frac{c^*}{(1-t)} [(1-P)U'(c^*+g^*) + PU'(c^*+\pi g^*)] = \lambda \frac{\partial R}{\partial t}
\]

and of course (10) and (11). \( \lambda \) is the Lagrange multiplier.

The case where \( PF = 1 \) (that is, discouraging evasion) and thus \( g^* = 0 \) is the possible local optimum to this problem. If another local optimum exists then we have to compare the actual levels of the
two utilities in the two optima. The fact that one can find a combination $(t^e, \pi^e, P^e)$ that increases welfare above the welfare that is achieved when $PF = 1$ should not surprise us. As is well known, for high $t$, $\frac{\partial R}{\partial t}$ may be negative. In this case, allowing tax evasion will have an effect similar to reducing the tax rate, and therefore increase welfare. But, in this case a preferred policy would be to directly reduce the tax rate. We have not been able to find a utility function where, when $\frac{\partial R}{\partial t} > 0$, allowing tax evasion increases welfare beyond what will be achieved by an optimal tax rate. We can say, though, that the two cases discussed by Weiss do not substantiate his claim.

One example, presented by Weiss is a utility function of the type $U(c, t) = \frac{1}{1-b} c^{1-b} + \delta t$. He shows that the condition for a welfare gain from tax evasion is $t > b$. However, in this case, the condition for Pareto efficiency ($\frac{\partial R}{\partial t} > 0$) is precisely $t < b$, which implies that tax evasion causes a welfare gain only in the situation where the tax rate is not an efficient one. $\text{12/}$

The other example presented by Weiss is the constant elasticity of substitution utility function $U = [\delta c^{-\rho} + (1-\delta) x^{-\rho}]^{-\frac{1}{\rho}}$. The existence of some combination of raising $t$ and reducing $P$ that creates a welfare gain over a tax with no evasion is ensured by $\text{13/}$

\begin{align*}
(18) \quad \frac{\rho}{1+\rho} &> \frac{1-t}{t} + \frac{\delta -1}{1-\delta} \left(1+\frac{1}{(1+\rho)} \right) \frac{1}{t} \left(1-t\right)\frac{(2+1)/(\rho+1)}{/(1+\rho)}. 
\end{align*}

Condition (18) implies that $\rho < 0$. In this case the indifference curves intersect both axes. However, along the axes the marginal
utility of income is constant, thus violating the assumption of risk-averse taxpayers. To see this, consider a taxpayer with a C.E.S. utility function and with $\rho < 0$ who has to solve the following problem

$$\max \ (1-P)[\delta(c+g)^{-\rho} + (1-\delta)c^{-\rho}]^{-\frac{1}{\rho}} + P[\delta(c-rg)^{-\rho} + (1-\delta)c^{-\rho}]^{-\frac{1}{\rho}}$$

subject to $c = (1-t)(1-l)$

The taxpayer can choose $l = 0$. Then he ends up with a linear utility function $(1-P)\delta(c+g) + P\delta(c-rg)$ and he would prefer to gamble as much as possible as long as $PF < 1$. This means that the first order conditions for a maximum are not useful because the corner solution will be preferred by the taxpayer.

We were unable to find any other examples that will confirm Weiss' contention. The question of whether there exists an example in which it is desirable to allow the taxpayer to cheat is still open. On the other hand it is shown that it is possible to have excess burden of tax evasion and the excess burden of the tax rate independent one on the other. Thus we can conclude that as a first approximation the excess burden of tax evasion and the excess burden of the tax rate may be added in order to estimate the excess burden of the tax.
NOTES


2. This formulation of the model is similar to Christiansen (1980).

3. For a discussion of the relation between the probability of detection and the penalty rate see Fishburn (1979) and Christiansen (1980). However, it is worth noting that the use of sampling probability of the IRS as a proxy for p (see for example, Maital (1982, p. 247)) underestimates the true probability of detection. Reports by a third party, e.g., the employer, increase the probability of being caught.

4. Where \[ \frac{\partial P}{\partial c} = P'(g, y) = 3g_y/3c \frac{P'(g, y)}{T'(x) - T'(y)} \leq 0. \]

5. These conclusions differ from those reached by Fencavel (1979) who was interested in the income (rather than tax) evaded.

6. Usually, the excess burden is defined in terms of income. Assuming that the marginal utility of income is known, enables us to translate the loss in utility to a loss of income.
7. As here defined, the excess burden of tax evasion is that due to uncertainty. In addition, the tax rate must be raised in order to offset the tax revenue lost through evasion, which increases the excess burden of the income tax. We prefer to allocate the elements of the burden in this way, since it is consistent with the case of evasion with a lump-sum tax.


9. The first-order approximation can be derived by a Taylor approximation of the utility function round $c + (1-PF)g$.

10. Weiss uses a tax evasion model in which the penalty rate is a function of the income evaded. This makes no difference in the present context.

11. Note that $\frac{3R}{3\pi} \bigg|_{PF = 1} = 0$.

12. It is worth noting that if the utility function is of the type $U(c) + \delta i$, then the condition for Pareto efficiency is $b > t$, where $b$ is the Arrow-Pratt relative risk aversion.

13. See Weiss, p. 1348.
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