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Cocoa Product Price Relationships

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SUMMARY

1. This study is an extension of the Division's earlier modelling of the world cocoa economy. It bridges a gap in the specification of cocoa bean demand and cocoa product price functions. The model of the determination of the prices of cocoa products (liquor, butter and powder) presented here takes into explicit account the economics of cocoa processing on the one hand and the demand for the individual cocoa products on the other. Previous analysis of this part of the cocoa market has been limited to deriving the ratio of each product's price to the bean price on the basis of the physical conversion rate. It is argued here that these conversion factors are themselves the result of economic decisions regarding the amount of cocoa fat to leave in cocoa cake and therefore these decisions should be treated as endogenous to the analysis of price determination in the cocoa market. In the specification developed here the prices of cocoa liquor, butter and powder are determined by the excess demand for each product and their expected profitability, measured as the difference between the "warranted" price (or the product of the bean price and the technical conversion factor) and the lagged product price.

2. In earlier studies, absence of data on production and consumption of cocoa products forced reliance on arbitrarily-determined conversion factors. Deficiencies in the quality of product price data are to be expected since a large proportion of product use is in vertically-integrated operations and there exists substantial heterogeneity in the products. Cocoa product use and output data have been generated for this study econometrically, using auxiliary observed relationships which are based on technical, albeit imperfect, information. One test of the robustness of this data is in how well the product price and new cocoa grindings equations fit historical data. The results suggest that the data are robust.

3. From the estimation of the model developed here, the price elasticity of bean grindings, a proxy for world consumption of beans, is found to be -0.4 compared with estimates of about -0.25 from analysis in which product price relationships are omitted. A larger elasticity estimate was expected since beans are ground not only in response to signals from the end-product market but also in response to changes in the relative profitability of using cocoa fat instead of other vegetable fats. In turn, cocoa product demand (a component of total foodstuffs) is essentially driven by income and population growth. At the world level, income and price elasticities of cocoa product demand are bound to be small as is expected of most food items. However, the fact that other vegetable fats can substitute for cocoa fats increases the price elasticity. The demand for cocoa is the most inelastic (-0.06) of the cocoa products because it faces the least competition and because it represents only a small share of the total intermediate product costs.

4. Estimation of the cocoa product price model indicates that the adjustment of prices to their warranted level is not instantaneous and that, in fact, product prices are seldom a fixed (technical conversion factor) multiple of cocoa bean prices as implied by those models which omit the cocoa products sector. For example, the cocoa liquor price adjustment parameter is

less than unity (0.84) and the liquor price is related to the bean price by a factor of 1.11, not by the technical conversion factor of 1.25. This finding has an important implication as the cocoa product-cocoa bean price ratio is often used as a guide in assessing the costs-benefits of processing cocoa products versus exporting raw cocoa beans, as well as the choice of the processing technology. The speed of the adjustment of cocoa butter prices to their warranted level varies positively with the size of cocoa bean stocks available for pressing. The average adjustment parameter is 0.6. But when cocoa beans were relatively scarce as in 1977, it decreased to 0.32--suggesting a slow adjustment of cocoa butter prices to their warranted level. The fastest adjustment occurred in 1965, which is also the year the ratio of cocoa bean stocks to grindings reached its highest level. The adjustment coefficient was close to 1.0 and the bean stocks to grindings ratio was 0.79.

5. Output from the model has been used as input to activity analysis carried out to assess the profitability of downstream processing of cocoa beans by producing countries and to derive optimal processing technology. From this analysis, it was found that choice of the optimal technology (hydraulic press, expeller, expeller and solvent extraction are the technologies presently in use) depends on bean and product prices as well as on the strength of demand for the various products.

6. After the technological and other events that around 1977 led to a large increase in the direct use of cocoa liquor in chocolate confectionary--as well as to the demand for high-fat cocoa powders--the basic cocoa product price ratios vis-a-vis cocoa beans changed substantially. It is interesting to examine the question of choice of processing technology against this background of substantial changes in price relationship. Using the results of the cocoa product demand estimations and prices prevailing during the period 1980-83, the optimization model results show that processers would be advised not to confine themselves to a single processing method. They should be able to convert beans directly into butter through expelling and solvent extractions as well as to be able to press for butter, and cake.

7. As regards investment in cocoa processing by the cocoa bean producing countries, given their current difficulties of competing in the very heterogeneous cocoa powder market, it seems more profitable to opt for investment in processing technologies whose joint product mixes are weighted in favor of output of the more homogenous cocoa butter.

I. INTRODUCTION *

8. The primary motivation for extending the model of the cocoa market to include the determination of prices of cocoa products--cocoa liquor or mass, cocoa butter and cocoa solids (i.e. presscake/powder)--was the recognition that the cocoa market has in fact four distinct submarkets (beans, liquor, butter and cocoa solids). These submarkets are intrinsically linked. By modeling explicitly these interlinked markets it was hoped to develop price relationships between cocoa product prices and bean prices that take into account factors other than the technical fat conversion factors, which has been the practice hitherto.

9. A second motivation was that without explicitly modeling the interrelationships between cocoa product markets and the market for beans it is not possible to analyze the economics of cocoa processing, as, for example, in the determination of the optimal processing technology under different product prices, or the comparative costs of processing at the origin versus processing in consuming countries.

*The authors wish to express sincere gratitude to Ronald C. Duncan and Takamasa Akiyama for their comments and suggestions. The authors are solely responsible for any remaining errors and shortcomings.

10. A third major reason for undertaking an analysis of the interrelationships between bean and product markets was the increasing interest in processing in bean-producing countries--exclusively developing countries. The composition of cocoa trade has changed over the years from exclusively bean exports to a mixture of beans and product exports. In the 1962-65 period, producing countries were on average grinding only 16% of their production locally. In 1985, they ground 31% of total bean production.

11. The remainder of the paper is organized as follows: Chapter II underscores the relevance of this analysis of the cocoa product markets by describing the evolution of trade in cocoa products and evaluating the prospects for cocoa processing and exports of cocoa products by bean-producing countries. Chapter III provides the background for the specification of the model by describing the products and their processing technologies. Chapter IV presents the model for estimating supply and demand relationships for cocoa products and discusses problems related to the data. Since product price formation is an important subject in this study, it is necessary to discuss the theoretical underpinnings of the pricing mechanisms involved and the product price models suggested. This is done in Chapter V. The results from estimating the model are presented in Chapter VI together with recommendations with respect to processing in bean-producing countries, and the conclusions are presented in Chapter VII.

II. COCOA PRODUCT TRADE

Evolution of Trade in Cocoa Products

12. At one time, cocoa exports were exclusively in the form of beans with the trade flowing from producing countries (all of them developing countries in the tropical areas) to consuming countries mainly in the northern hemisphere. The structure of cocoa trade has undergone substantial changes with the increase of processing in cocoa bean-producing countries and the resulting increase in the share of trade in the form of processed products. In the middle of this century, only 11% of cocoa produced was ground by producing countries. Their share of grindings increased steadily to 34% in 1978/79 and have stabilized at that level. Since 1977/78, almost all cocoa paste sold on international markets has been processed in bean-producing countries. The bean-producers' shares of the markets for cocoa butter and powder/cake have also increased, though not to the same extent. Table 1 reports the evolution of cocoa products trade in terms of the shares of bean-producing countries.

13. The figures of Table 1 mask spectacular increases in the export shares by some countries that deserve acknowledgment. Brazil has achieved the most outstanding increases in grindings and exports of cocoa products. It has become a leading exporter of cocoa butter, ranking second behind the Netherlands. In 1985, Brazil exported 48,000 tons of cocoa butter and 36,000 tons of cocoa powder/cake--by comparison with less than 10,000 tons of exports of both products two decades earlier. Côte d'Ivoire has more than doubled its grinding capacity in less than a decade, and output increased from 40,000 tons

grinding capacity in less than a decade, and output increased from 40,000 tons in 1977 to 90,000 tons in 1985; Malaysia has increased its grinding output ten-fold to 20,000 tons over the same time span.

TABLE 1: WORLD SHARES OF COCOA GRINDINGS AND COCOA PRODUCT EXPORTS FROM BEAN-PRODUCING COUNTRIES

(PERCENTAGES)

	GRINDINGS	EXPORTS		
		COCOA PASTE	COCOA BUTTER	COCOA CAKE
1948/52	11	NA	35	
1958/62	16	NA	30	20
1968/72	21	57	53	30
1973/77	25	86	49	39
1977/78	32	92	48	48
1978/79	34	95	48	48
1979/80	33	93	46	46
1980/86	33	NA	NA	NA

NA: Not available.

SOURCE: DERIVED FROM GILL AND DUFFUS AND ICCO.

Prospects for Cocoa Products Trade

14. Exports of cocoa products by developing countries are expected to grow because of the following changing trade practices:

- i. Smaller manufacturers of products using cocoa in industrial countries prefer to buy the intermediate cocoa products instead of grinding their own, in order to have

grinding of cocoa beans for cocoa butter--to be used subsequently for chocolate manufacturing--may yield an unwanted amount of cocoa powder; this is avoided if the manufacturer imports the specific product quantity needed.

- ii. On the supply side, recent studies [UNCTAD (1984), Karunasekera (1983)] suggest that there are potential gains for developing countries in processing some--but not necessarily all--cocoa products. Many cocoa-producing countries have been contemplating increases in processing of cocoa beans to increase domestic value-added.

15. When considering the past increases in cocoa processing in the bean-producing countries and the potential for further increases, it is important to note that over this historical period the advent of the Lomé Convention (between the EEC and a group of African, Caribbean and Pacific countries) and the GSP (Generalized Scheme of Preferences for exports of developing countries to industrial countries) provided incentives for processing in the developing countries. As a result of these international agreements, the barriers against imports of cocoa products are now low to zero in most industrial countries. It is noteworthy that the response to this lowering of barriers was quite low among the African producers, which suggests some constraints on the supply side. Another qualification to be observed when drawing conclusions from the past increases in processing activities in developing countries is that Brazil and Ecuador provided subsidies to artificially encourage this

activity. Such policies are economically costly and are not to be encouraged.

1/ It is not known to what extent differential tax policies have led to the movement of processing activities from industrial to developing countries. Multinational companies are heavily involved in cocoa processing activities and their investment activities are sensitive to differential tax rates between countries.

1/ This fact is best illustrated by the Ecuador cocoa industry as recounted by Aronson (1987) an Ecuadorian cocoa exporter: "In the mid [1970s] the Ecuadorian Government, through a complex of subsidies which totalled 40%, encouraged the development of a local liquor industry which, at its peak around 1978, reached a grinding capacity of about 150% of the total crop. This excess capacity created several problems: (i) Fierce competition on the part of the firms to place their product in a world market that was not ready for such ample supplies of Ecuadorian liquor, which in effect amounted to a transfer of the subsidy to the [foreign] consumer; (ii) A local price squeeze, which drove up the price of beans internally to levels at which bean exporters could no longer export at market prices. The result of this situation was a long period of conflict between industry, government, exporters and producers in which everyone came out losers."

III. COCOA PRODUCTS: PRICE FORMATION, SUPPLY AND DEMAND

16. An understanding of the model developed in the subsequent chapter requires a description of processing activities and the historical price patterns for cocoa products, as well as the technical conversion factors important for product output data estimation. Prior to this a description of the various cocoa products is in order.

Cocoa Products and their Characteristics

17. Cocoa, also referred to as cacao, is the common appellation for a tropical tree called Theobroma cacao. ^{1/} When the fruit (an American football shaped pod) is harvested, the seeds are removed to undergo fermentation and drying. Fermentation may be done in wooden buckets covered with banana leaves. Sometimes the pulp-encased seeds are simply wrapped in the banana leaves. After fermentation, the beans are sun dried on a cement-covered area. Besides making handling easier, the drying of cocoa beans improves their aroma and reduces their mildly bitter taste.

18. Fermentation and drying are the only two processes performed at the farm level. Further development of flavor is achieved at the industrial level by roasting which also reduces moisture and acidity. The roasted beans are

^{1/} This name was given in the 18th century by Carolus Linnaeus the Swedish botanist, originator of a system of taxonomic classification. It means food of the gods. Cocoa was at that time the most honored beverage of the highest class in the Montezuma era. The tropical beverage was spread to Europe by the Spanish navigator/explorer Hernando Cortes.

more friable and ready to be ground after the nibs or broken pieces of kernels have been isolated from the shell husks and germs.

19. The first product obtained from grinding cocoa nibs is called cocoa liquor. It is a heavy liquid that contains 53% to 56% fat. Cocoa liquor is also referred to as chocolate liquor or cocoa mass. In this form the product can be used for making chocolate for baking, or undergo further grinding for manufacturing butter and powder.

20. Cocoa butter is the solid fat derived from cocoa liquor either by grinding, hot pressing or by other chemical processing. It is used in the manufacture of chocolate, in the cosmetic industry and for pharmaceutical preparations such as suppositories.

21. Cocoa powder is obtained by pulverizing cocoa cake, the de-fatted solid product left after butter has been extracted from the liquor. Cocoa powder contains butter residues in varying proportions: usually between 10% and 20%. 1/

22. During the past decade, there has been an increasing use of cocoa product substitutes in the confectionary industry. To understand the reason for their popularity, a little knowledge of the physical properties of the main ingredients in chocolate confectionery is necessary.

1/ Beside cocoa powder, cocoa cake also produces theobromine, an alkaloid ($C_7H_8N_2O_2$) which is a natural nerve stimulant.

23. Chocolate confectionaries have two distinctive characteristics: their aroma and their ability to dissolve. The aroma is provided by the solid products derived from grinding cocoa beans: cocoa mass and cocoa powder. The attractive property of chocolate products of remaining solid at room temperature but melting when placed in the mouth is due to cocoa butter, the fatty cocoa derivative. While it has been difficult to find a non-cocoa product that could give chocolate its characteristic flavor, some limited success has been achieved in using non-cocoa vegetable fats in chocolate confectionary.

24. There are virtually no substitutes for cocoa powder, although during the 1977-79 period of record high cocoa prices some manufacturers tried various powder substitutes derived from carob. In France, scientists have been able to isolate from coal the type of enzyme responsible for the chocolate flavor. It is not known whether the discovery has crossed the laboratory boundary, but current price prospects do not justify such an investment.

25. Competition is stronger, however, for cocoa butter from a still marginal but increasing use of different types of vegetable fats. There are three types of fats that are competing with cocoa butter in confectionary manufacture. These are cocoa butter equivalents, replacers and improvers.

26. **Cocoa Butter Equivalents (CBEs):** These products have the property of being compatible with cocoa butter; thus they can be complementary to cocoa butter when manufacturers are trying to reduce the total cost of fat ingredients. Non-lauric, hardened fats constitute most of the CBEs.

27. **Cocoa Butter Replacers (CBRs):** These products are intended to replace cocoa butter use in confectionary. Since the CBRs are usually not compatible with cocoa butter they are blended with cocoa powder to meet full flavoring requirements. Butter substitutes are obtained by two processes of refining vegetable oils: fractionation and hydrogenation. The first process (one that yields a high-quality butter substitute) is currently the most expensive. It decomposes the vegetable fat into fractions in order to obtain specific physical properties.

28. **Cocoa Butter Improvers (CBIs):** This last category consists of vegetable fats that perform better than cocoa butter in the conservation and handling of chocolate confectionaries. They have a higher melting point than cocoa butter; thus they have a longer shelf life and are more suitable for manufacturing chocolate to be marketed in tropical regions.

Processing of Cocoa Products

29. Cocoa beans are transformed through several processes into intermediate inputs--cocoa liquor, cocoa butter, cocoa cake/powder--that are the raw materials for the chocolate confectionary and chocolate flavors industry. After an initial stage of cleaning, roasting (unless the beans are to be expelled solely for cocoa butter) and winnowing to separate the nibs from the bean shells, the nibs are fed into grinders to produce cocoa liquor (also referred to as chocolate mass or cocoa paste). The liquor is then pressed hydraulically to produce cocoa butter and presscake which is subsequently

ground to produce cocoa powder. In the hydraulic press the cocoa butter content of the liquor is reduced from approximately 55% to a cocoa presscake which may have a fat content ranging from 12-25% according to the final applications of cocoa powder. An alternative route to cocoa butter and cake adopts expelling or extrusion of cocoa nibs (or even beans themselves to recover the fat content of the shells) without going through liquor production. Under expeller pressing it is possible to reduce the fat content of cocoa cake to 8-9% without great difficulty and so a relatively higher proportion of butter can be recovered. The flavor of cocoa butter obtained in this way is mild and in this respect is quite different from that obtained through hydraulic pressing. Expeller pressing is often used to extract the fat from sub-standard beans. Another way to extract cocoa butter is through solvent extraction from expeller cake or cocoa or chocolate residues. Solvent-extracted butter needs to be subjected to a refining process. A schematic representation of the various technologies described above is to be found in Figure 1.

30. In general, the choice of the technology of processing depends on the relationship between factor costs in each country. Hydraulic pressing in its various forms is the most widely-used process, especially in developing countries where capital is scarce (in those countries processes (6) and (7) are most prevalent). Nib and liquor alkalinization (or the Dutch process) is a more capital-intensive method that yields deeper-colored cocoa products.

31. The grinding of presscake produced from cocoa liquor results in a range of good quality powders. Compared to cocoa butter, cocoa powder is very

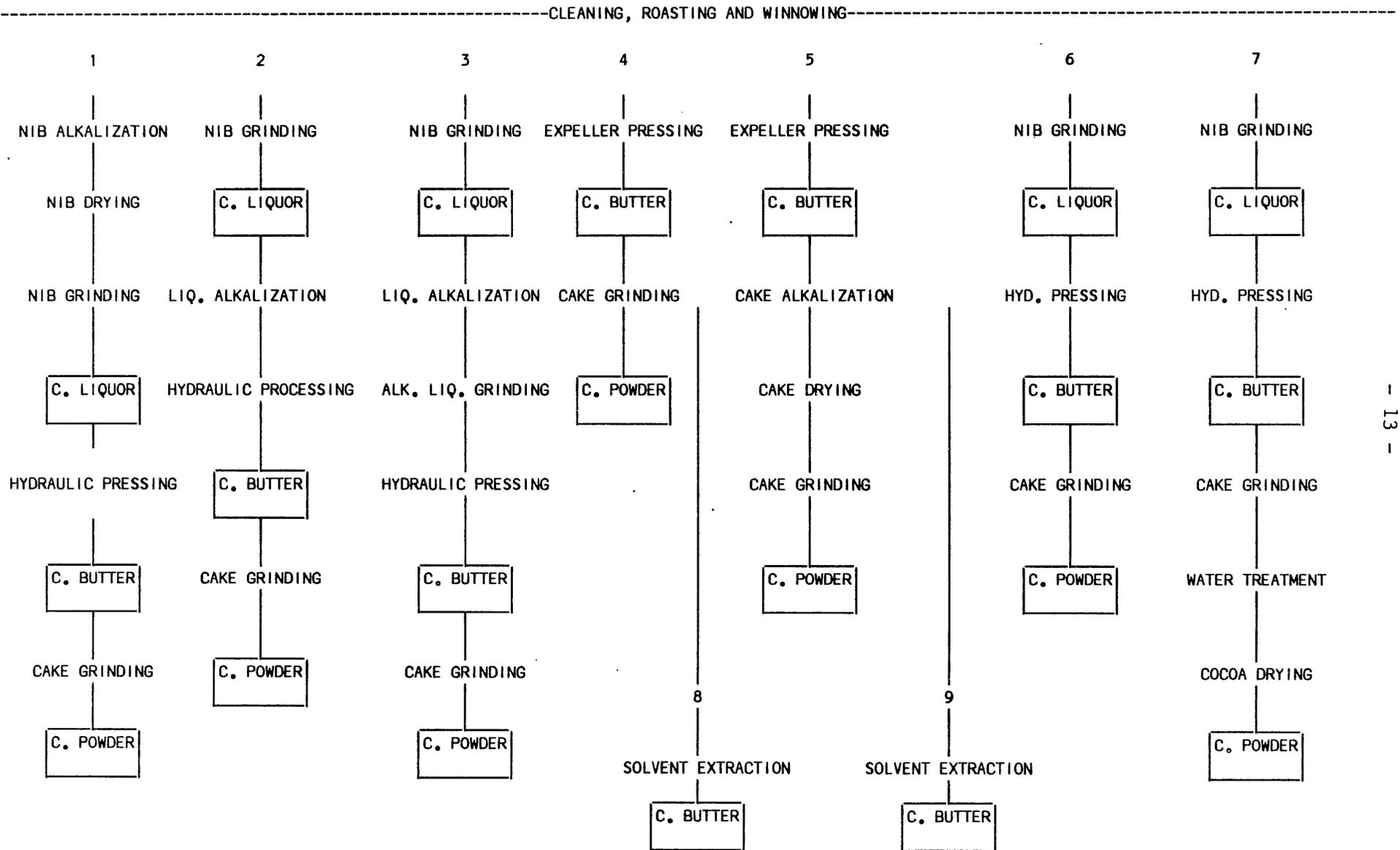
much a heterogenous product differentiated by fat content, fineness of grind, color and flavor. The powder produced from expeller cake has, in general, inferior flavor and is also difficult to obtain in a very fine form. In practice, therefore, the final outlets for powders produced from expeller cake are more limited.

Technical Rates of Conversion from Beans to Products

32. In the specification of production relationships for cocoa products in the next chapter, extensive use is made of technical conversion factors. It is appropriate at this stage to review their derivation. At the initial stage of processing, liquor or paste is produced from the beans: the liquor content of 100 tons of beans is approximately 80 tons. The physical composition of liquor is not precisely fixed, but the conventional view, based on processing experience, is that some 55% of liquor is fat and the other 45% cocoa solids.

33. Thus, in an ideal situation, 100 tons of beans or 80 tons of liquor can yield 44 tons of fat or cocoa butter and 36 tons of cocoa solids. Generally for cost reasons, mechanical pressing does not recover all butter and defatted cake nor cocoa solids. Depending upon the choice of pressing equipment, the pressed cake contains fat ranging from 6-8% up to 22-24%. Liquor pressing is thus a spectrum of technologies yielding different quantities of butter and cocoa presscake and it is part of the processing decision to choose how much butter to recover and what fat to leave in the cake.

FIGURE 1: A SCHEMATIC REPRESENTATION OF NINE COCOA PROCESSING TECHNOLOGIES



SOURCES: ADAPTED FROM MINIFIE, P. 42.

34. Therefore, contrary to the impression given by the use, say, of FAO conversion factors for processing beans to butter and to powder, these factors do not reflect absolute physical conversion rates. In fact, they represent the average conversion rates in a specific situation: if the bulk of world presscake produced is either 10-12% fat content or 22-24% fat content and if the production of low-fat cake to high-fat cake is in the ratio 2:1, it follows that on average presscake contains 15% fat. Given that 80 tons of liquor contains 44 tons of butter and 36 tons of cocoa cake, if the cake contains 15% fat then 80 tons of liquor will produce 42.4 tons of presscake [$\approx 36 \div (1-.15)$] and 37.6 tons of cocoa butter [$\approx 80-42.4$]. The conversion factors reported by FAO result from supposing that of the 100 tons of beans producing 80 tons of liquor, half are devoted to each of the two joint products--cocoa butter and cocoa cake. In that case the following conversion factors result:

$$\text{cocoa butter} = 50/37.6 = 1.33$$

$$\text{cocoa cake} = 50/42.4 = 1.18$$

35. Under this same assumption (i.e. on average, cake contains 15% fat), we can easily derive the technical input-output relationships.

$$\text{butter production} = \frac{37.6}{80} (=0.47) \times (\text{Liquor available for pressing})$$

$$\text{cake production} = \frac{42.4}{80} (=0.53) \times (\text{Liquor available for pressing})$$

The factors 0.47 and 0.53 are used in the production relationships specified in Chapter IV.

36. The foregoing suggests two observations. In the first place it explains why the ratio between prices of cocoa products and prices of beans does not remain constant. The existence of joint products and the wide range for the fat composition of cocoa cake means that a given price for beans is inadequate to determine the values given to the two essential components of cocoa beans--the fat which constitutes cocoa butter and the "flavor" which is what imparts value to cocoa cake.

37. The second observation is that while it is true that production relationships display fixed proportions, the processes can in fact alter the proportions in which butter and cake are produced from a given quantity of liquor by varying the fat content of cake. For example, if we limit the fat content of cake to vary between zero and 24%, the production possibility frontier for cocoa butter and cake from 80 tons of liquor, is then described mathematically by:

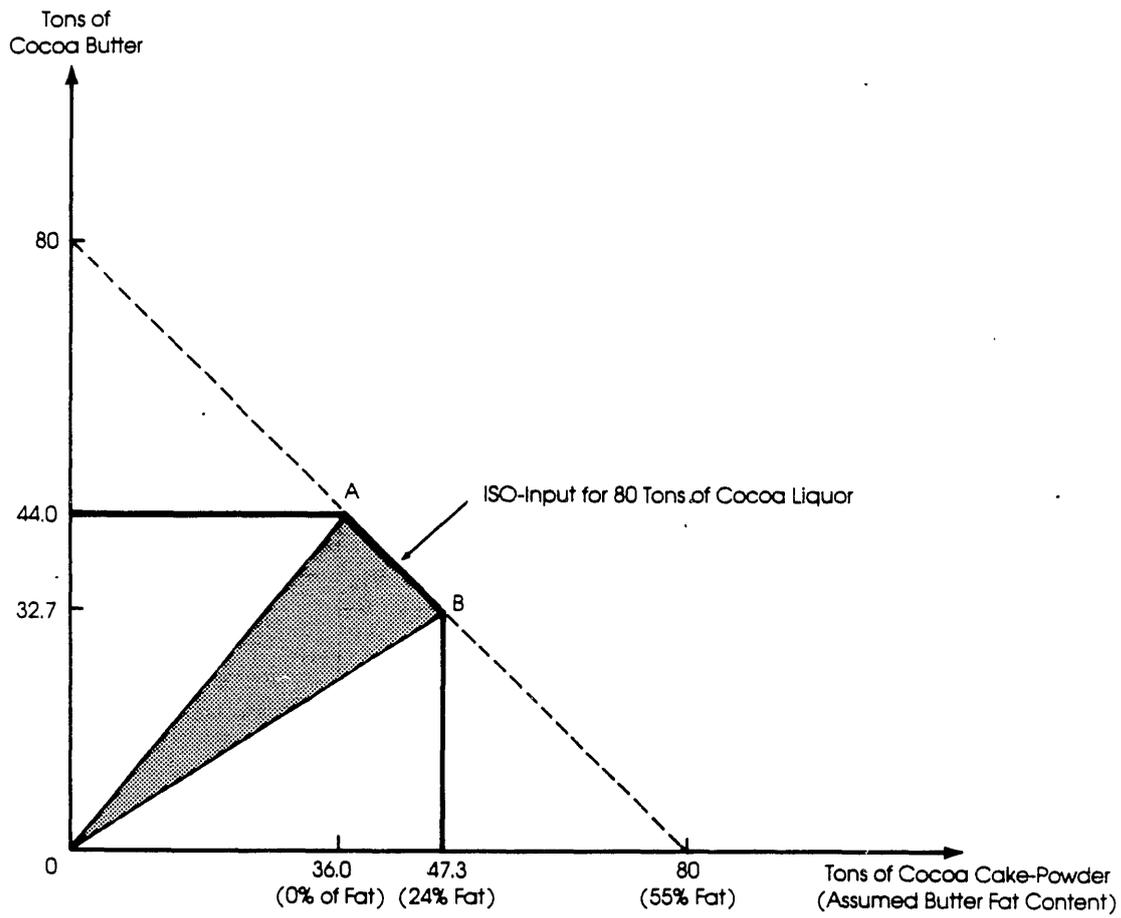
$$\text{BTPD} + \text{PYPD} \leq 80$$

$$\text{BTPD} \leq 44$$

$$\text{PYPD} \leq 47.3$$

(where BTPD is butter production, PYPD is powder production) and graphically as shown in Figure 2. In Figure 2, lines from the origin 0

Figure 2: Butter and Cake-Powder Production Possibility Frontiere



which intersect the segment AB 1/ describe alternative liquor pressing technologies: each line gives the proportions in which butter and cake (of varying fat content) are obtained.

38. Finally, an intuitive interpretation of the relative valuation of cocoa products can be given along the following lines. Supply and demand, not of the quantity of beans but of the quantities of "fat" and "flavor", determines the value of cocoa beans. Since liquor contains both and requires 1.25 tons of ground beans per ton of liquor, liquor values should be in the ratio 1.25 times the bean price. Once we turn to the joint products--cocoa butter and cocoa cake/powder--we find that their composition in terms of "fat" and "flavor" are very different from that of the cocoa bean. The valuation of the products by the market indicates that "fat" is more highly valued than "flavor". If the conventional FAO factors were to be a guide to the market's valuation of "fat" and "flavor", we should observe butter and powder/cake being priced at 1.33 and 1.18, respectively, times the bean price. Actual product prices show that cocoa butter is valued substantially higher than this; so much so as to recover almost all the processing cost from butter alone and to treat cake/powder as a by-product of low value. Only at the time of the severe bean shortage of 1977/78 has the cake/bean price ratio assumed importance (see Table 2) and this event is generally explained as the result of a significant increase in demand for powder in confectionary and bakery uses. The model results presented below suggest that the 1977/78 butter/bean

1/ The segment AB intersects the cake production axis at 80, i.e., liquor can be viewed as cake containing 55% fat.

TABLE 2: COCOA PRODUCT PRICES AND THEIR RATIOS TO BEAN PRICES

	COCOA LIQUOR		COCOA BUTTER		COCOA CAKE/POWDER		GPM
	PRICE (\$/TON)	LIQUOR/BEAN PRICE RATIO	PRICE (\$/TON)	BUTTER/BEAN PRICE RATIO	RATIO (\$/TON)	POWDER/BEAN PRICE RATIO	RATIO*
1961	503.7	1.04	1,052.6	2.17	347.0	0.71	2.88
1962	421.5	0.92	1,125.4	2.45	314.8	0.68	3.13
1963	350.7	0.63	1,198.4	2.17	283.2	0.51	2.67
1964	445.9	0.88	1,179.0	2.33	296.1	0.58	2.92
1965	319.4	0.87	1,007.8	2.75	265.3	0.72	3.47
1966	350.1	0.67	1,011.7	1.95	241.4	0.47	2.41
1967	424.9	0.71	1,221.3	2.04	285.2	0.47	2.52
1968	546.9	0.76	1,440.9	2.00	308.2	0.43	2.42
1969	840.8	0.93	1,900.8	2.10	363.8	0.40	2.50
1970	866.7	1.28	1,618.2	2.40	429.5	0.64	3.03
1971	704.2	1.31	1,296.0	2.41	447.3	0.83	3.24
1972	710.4	1.10	1,309.2	2.03	423.9	0.66	2.69
1973	1,101.4	0.97	2,106.8	1.86	480.6	0.42	2.28
1974	1,941.5	1.24	3,360.3	2.15	658.5	0.42	2.57
1975	1,836.1	1.47	3,239.0	2.00	848.4	0.68	3.28
1976	2,057.9	1.00	3,351.2	1.64	1,005.6	0.49	2.12
1977	4,212.9	1.11	5,137.9	1.35	2,968.3	0.78	2.13
1978	4,296.5	1.26	4,763.2	1.40	4,689.1	1.38	2.77
1979	4,149.9	1.26	5,462.5	1.66	3,925.7	1.19	2.85
1980	3,466.9	1.33	6,293.2	2.42	2,229.0	0.86	3.27
1981	2,532.1	1.22	5,042.3	2.42	1,173.2	0.56	2.98
1982	2,258.4	1.30	4,551.0	2.62	1,067.4	0.61	3.23
1983	2,111.0	0.99	4,013.6	1.89	1,046.7	0.49	2.38
1984	2,200.0	0.92	4,200.0	1.75	1,050.1	0.44	2.19

* GROSS PROCESSING MARGIN RATIO, OR THE SUM OF BUTTER AND POWDER PRICE RATIOS.

SOURCES: FAO AND IECCM.

and cake/bean price ratios are more convincingly explained by a shift in liquor processing towards recovering more butter from liquor because of the bean shortfall coinciding with relatively firm but not exceptional demand for high fat content powders. Subsequently, the lagged adjustment in liquor processing in the period after 1978 toward producing the high fat content powders resulted in high butter/bean price ratios that persisted through 1981/82, even though bean supplies were abundant.

Historical Product Price Behavior

39. During the 1960's, cocoa liquor was priced below cocoa beans not only in bean equivalent but also in absolute terms. Indeed, as Table 2 shows, the liquor to bean price ratios averaged 0.79 over the period 1962-69, with a low of 0.63--showing that liquor, the product, was priced at only 63% of the cost of cocoa beans. The bean equivalent break-even price ratio--which is the technical conversion factor--is 1.25. There are two plausible explanations for such product price "backwardation". One is the alleged transfer pricing practiced by the vertically integrated industries controlling over 60% of cocoa liquor produced during the 1960s, whereas beans were produced and sold by more numerous agents. The second explanation, which is related to the first mentioned, has to do with the fact that since the main purpose of grinding beans was to extract butter, a small share of liquor produced was traded for its own value. Thus a thin market situation prevailed in which price did not reflect the "real" input costs (bean prices) of producing liquor.

40. The 1960s also saw a relatively high butter to bean price ratio (averaging 2.22) which compensated for the low liquor to bean price ratio. The low powder to bean price ratio (averaging 0.53) during the same period is also consistent with the fact that cocoa butter fat was the only valued output of the cocoa bean grinding process.

41. As the share of liquor produced outside vertically-integrated operations increased in the 1970s (see Table 1), liquor prices tended to reflect more closely the technical conversion ratios between the products (see Table 2). The liquor price ratio exceeded unity during the period.

42. The sum of butter/bean and powder/bean price ratios (a measure of gross processing margins; GPM ratio in Table 2) has always been greater than 2.53--the sum of the technical conversion ratios for both products--except during 1976-77, the period of record-high cocoa bean prices and the slow product price adjustments.

43. From examination of the cocoa products' price ratios (see Table 2) it is evident that cocoa butter has traditionally commanded a considerable premium over the bean price. The converse has been true for powder/cake except for the period 1977-79. The most important factor underlying the valuation of cocoa products is the fat content: it is the fat in cocoa products which holds crucial significance for chocolate confectioners. That this is the case is further demonstrated by the fact that cocoa powders with higher fat content, especially when they are alkalized, command prices that are at a considerable premium to the bean price. A spectrum of cocoa powder prices and price ratios is presented in Table 3.

TABLE 3: PRICES OF COCOA POWDERS AND POWDER/BEAN PRICE RATIOS BY BUTTERFAT CONTENT AND ALKALIZATION PROCESS, NOVEMBER 28, 1986

Cocoa Powder	Price (¢/lb)	Powder/Bean Price Ratio*
10-12% Natural Butterfat	48- 53¢/LB	0.55
10-12% Alkalized	58- 63¢/LB	0.65
10-12% Red Alkalized	79- 89¢/LB	0.91
10-12% Black Alkalized	89- 99¢/LB	1.02
16-18% Natural Butterfat	60- 65¢/LB	0.68
16-18% Red Alkalized	94- 104¢/LB	1.07
22-24% Butterfat	72- 77¢/LB	0.81
22-24% Red Alkalized	106- 116¢/LB	1.20

* As of November, 1986 when the bean price was ¢91.8/LB

Sources: Milling and Baking News, and IECCM.

44. It is relevant to note that while cocoa butter is a more or less homogeneous product, cocoa powder is a highly heterogeneous product, differentiated by fat content, fineness of grind, color and flavor. The main outlets for cocoa powders are in the beverage, dairy and baking industries. Another major use of cocoa powder is in the coating of some chocolate-flavored dairy products. Accordingly, the low-quality, high-volume cocoa powder or cake obtains quite low prices (relative to bean prices), since this product is abundant relative to its outlets in the bakery and dairy product sectors. A very small amount of the cocoa powders used in chocolate flavoring of bakery and dairy products will suffice to impart the desired flavor. At the other extreme are premium cocoa powders, produced to specification as special chocolate flavoring agents, which command very high prices appropriate to a highly-differentiated product.

45. The preceding points can be illustrated from the behavior of the price data for cocoa beans, butter and powder in Table 2. From 1975 to 1977, prices of cocoa powder more than tripled and in 1978 cocoa powder was priced almost as high as cocoa butter. Having presumably a more inelastic demand (one reason is the absence of powder substitutes), cocoa powder experienced a far greater price increase than cocoa butter as a result of the 1976-1978 cocoa bean shortage. The major reason for the four-fold increase of powder prices over the above period was due to the large increase in the demand for high fat powder following a shortfall in the supply of cocoa butter. This price movement disturbed the traditional price relationship in the cocoa industry and hence affected the relative profitability of the derived and raw products. Converters could be expected to give preference in their grindings to powder production rather than to butter during that period. However, there could not be a dramatic substitution in production because powder and butter are produced in technologically-fixed proportions (although by changing from pressing to solvent extraction it is possible to change these proportions to some extent). Industry sources assert that grinders in such a situation would typically lower their bid on one product (in this case cocoa butter) as long as they could compensate through the other product. (See Annex A for a graphical explanation.)

IV. SPECIFICATION OF THE COCOA PRODUCTS MODEL

46. One of the main uses of the model is to provide the basis for forecasting cocoa product prices. Although the known differences in income elasticities between countries argue for the modeling of supply and demand by region, such an approach is not feasible due to lack of reliable regional data of the demand and supply of cocoa products.

Cocoa Market Relationships

47. A simplified flowchart of global cocoa market interactions is schematized in Annex B. The core entities that are the subject of most cocoa models are the supply and demand for beans and bean prices. Inventories of cocoa beans have been found to play a key role in short-term price formation (Weymar). Export strategies such as scheduled sales by non-atomistic marketing boards can also affect cocoa prices (Gbetibouo). Income and population, the usual demand shifters, affect the quantity of cocoa beans consumed, not directly, but via the demand for end products such as chocolate confectionary and non-confectionary cocoa-based goods. The logical link between the cocoa bean block and the exogenous end product block is the intermediate product

market. 1/ Prices of intermediate products are determined by output and the demand for grindings. The quantities of intermediate products depend on the supply of beans and the technology of grinding.

Structure of the New Cocoa Model

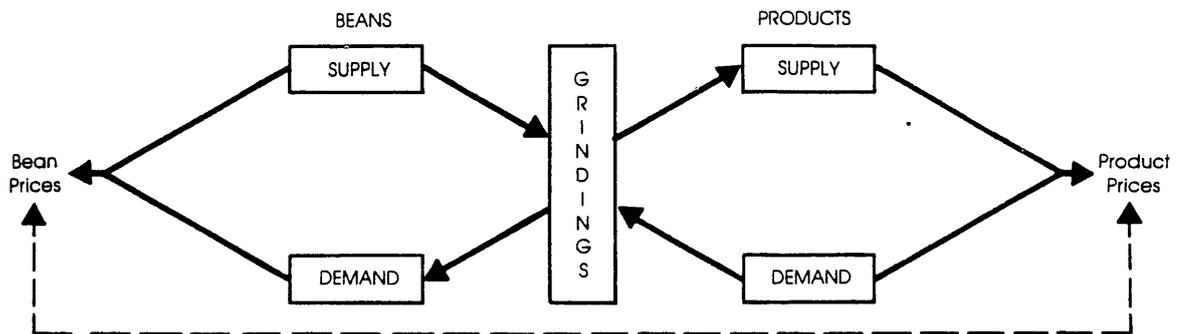
48. The cocoa products' model builds on the Division's cocoa bean model. 2/ In effect, the new cocoa model consists of the cocoa products model described in this paper together with endogenous bean supply equations taken from the cocoa bean model. Figure 3 demonstrates how the linkages between the two blocks are made. A more comprehensive representation of the linkages is displayed in Annex E.

49. The supply of beans affects the availability of cocoa products via the technical coefficients of converting beans into products. The demand for products and the supply of beans jointly determine the prices of the products which, in turn, affect the profitability of grinding. The expected level of grindings is one of the major determinants of the current cocoa bean price. With relevant lags, the price of cocoa beans will affect the production capacity and supply of cocoa beans in each exporting country as specified in the bean submodel.

1/ The next step for a comprehensive cocoa model would be to endogenize the end-product market. In the current specification this market is assumed to be essentially driven by population and income; the substitution effects of non-chocolate confectionaries are ignored.

2/ See Akiyama and Duncan (1982) and Akiyama and Bowers (1984) for a description of the bean model

Figure 3: Cocoa Submodels' Linkage



43. The processing activity has been described in Chapter III. The economics of processing as it relates to products' price formation is discussed in subsequent sections pertaining to the products' price equations. Since prices in the model emerge as the result of the adjustment behavior of product manufacturers to excess demand and to processing margins, it is necessary to discuss the supply and demand relationships for each product. Although product prices in the model are not strictly derived by equating supply to demand, a discussion of these relationships is necessary in order to comprehend the market forces that determine the excess demands for the products and to justify the chosen proxies for excess demand.

44. Another important relationship is the grindings equation. The grindings equation links beans to products through the physical transformation of the beans (the supply side) as well as via the expected profitability or processing margins (the demand side). The demand for grindings, which is an

important determinant of grindings, is estimated here as the sum of the individual product demands.

52. The levels of consumption of these products (liquor, butter and powder) are determined from input demand schedules. These input demands are expressed by chocolate and cocoa-flavored goods manufacturers. Theoretically, the demand for the three cocoa products should be expressed as:

$$PDQ_i = f (PP_i, PO, PS, Z)$$

where: PDQ_i = quantity demanded of intermediate cocoa product i
 PP_i = intermediate cocoa product i price
 PO = output price
 PS = prices of substitute inputs
 Z = exogenous shifters

53. The difficulty of getting data for some price variables--for instance, prices of CBE and CBS in the case of cocoa butter demand, and of the intermediate input in the case of the heterogenous cocoa powder--forced us to express the stated input demand relations in an ad hoc manner. These demand expressions, along with other important building blocks in product price formation, are presented below.

Liquor Production and Demand

54. **Production:** Liquor production at the world level bears a fairly constant relationship to grindings. With existing technology, the quantity of liquor produced from one ton of pressed cocoa beans is 800 kg. Thus

$$LQPD = 0.8 * BNGR$$

where LQPD = world production of cocoa liquor

BNGR = world cocoa bean grindings

55. **Demand:** Two components of liquor demand need to be distinguished. One is the liquor directly used in flavoring confectionaries. The other component is the liquor pressed for the extraction of cocoa butter. The latter component is simply a residual found by subtracting from the total amount of liquor produced the quantity of liquor consumed as direct input by the chocolate and cocoa-flavored product industries. The direct use of liquor in chocolate and chocolate-flavored products is viewed as an input demanded in the production of these goods. Thus the following equation is suggested.

$$LQD = f(CHPD, CFLPD, BTCHPDR, LQCHPCR)$$

where

LQD = direct demand for liquor

CHPD = index of chocolate production

CFLPD = index of chocolate-flavored confectionary production

BTCHPDR = ratio of butter use in chocolate production

LQCHPCR = cocoa liquor to chocolate price ratio

The ratio expressing changes in the amount of cocoa butter used for chocolate production (BUCHPDR) captures the impact of a complementary input while the liquor to chocolate price ratio (LQCHPCR) picks up the price effect. The chocolate flavor production (CFLPD) is proxied by the ratio of biscuit to chocolate production. The level of intermediate consumption of cocoa liquor should be positively related to the variations of CFLPD. This formulation suggests that the joint estimation of liquor, butter and powder demands would lead to efficiency gains since any shocks in liquor demand are likely to be correlated with shocks in butter/powder demands.

Butter Production and Demand

56. **Production:** Cocoa butter production is the sum of butter obtained from the beans and the cocoa nibs. The yield coefficient for the recovery of butter through mechanical pressing of beans is 0.47 on the assumption that the average fat content of cocoa powder is 15%. Expeller butter is the relatively small proportion of butter production that is obtained directly from cocoa nibs by mechanical extrusion. Thus:

$$BTPD = 0.47 [LQAV] + EXBTPD$$

where BTPD = cocoa butter production

LQAV = liquor available

EXBTPD = expeller butter

57. **Demand:** Demand for cocoa butter is specified as an input demand function in the production of chocolate products.

$$BTDM = f(CHPD, CFLPD, BTCHPCR, BTFTPCR)$$

where:

BTDM = cocoa butter demand
BTCHPCR = cocoa butter to chocolate price ratio
BTFAPCR = cocoa butter to vegetable fats price ratio

Since chocolate confectionary is the main end use for cocoa butter the variable CFLPD can probably be dropped. Its contribution to the explanatory power of the equation is likely to be small and, moreover, omitting it may help in the identification of the BTDM equation from the LQDM equation.

58. Cocoa butter substitutes (CBS) and extenders (CBE) are playing an increasingly important role in chocolate production. In order to take account of the substitution of butter by such alternatives we have introduced a price term in the form of a ratio of the price of cocoa butter relative to the price of other vegetable fats. The second price term is the butter/chocolate ratio to estimate the own-price elasticity of the input demand for butter.

Powder Production and Demand

Production: Powder or presscake is produced jointly with butter in the proportion 53/47.

Thus $PWPD = 0.53 [LQAV]$.

where $PWPD =$ powder produced.

59. **Demand:** Demand for powder is specified as an input demand function in the production of chocolate-flavored products, the main outlet for powder.

Thus

$$PWDM = f(CFLQ, PWCHPCR)$$

where $PWDM =$ cocoa powder demand

$PWCHPCR =$ cocoa powder to chocolate price ratio

60. Cocoa powder is used in a wide range of chocolate confectionary for which it is difficult to obtain individual output indices. Since butter and powder are jointly produced, powder output is necessarily positively correlated with butter demand. An increase in powder output will lead to a fall in the powder price and to an increase in powder consumption. Thus, powder use is likely to vary positively with butter demand. As this relationship has been incorporated by using OLS instead of a systems estimator, thus not allowing for across-equation correlation with the error term of the BTB equation, the simultaneous equation bias in the estimation of powder and butter demand will not be corrected. The powder/chocolate price ratio picks up price influences. Given that there are no important substitutes for cocoa powder, a relative price variable is not included.

Grindings Equation

61. The cocoa grindings equation can be viewed as being derived from the constrained minimization of the cost function for an intermediate input. The grinder wants to minimize costs subject to a predetermined demand for grindings. 1/ Within this minimization framework, the optimal factor use or the quantity of beans ground will depend on the marginal physical product of beans. Assuming that butter is the only valued product of the grinding activity, this marginal physical product will be equal to the cocoa butter price multiplied by the fat content. The cost function can be represented as:

$$C = F + \text{BNPC} * \text{BNGR}$$

- where
- C = total costs
 - F = fixed costs
 - BNPC = price of cocoa beans
 - BNGR = grindings

In this simplified cost function, variable inputs other than cocoa beans are ignored. Thus the following production relationship is suggested:

$$\text{ACP} = \psi (Q_{\text{bn}})$$

- where
- ACP = aggregate cocoa product
 - Q_{bn} = quantity of cocoa beans

1/ Demand for grindings is "predetermined" in the sense that it is assumed that the grinder holds to a naive expectation about future grinding needs which is the sum of the observed individual product demands.

Recasting the constrained minimization problem, adding the Lagrangian term and solving for the first-order conditions yields:

$$C = F + \text{BNPC} * \text{BNGR} - \lambda [\psi(Q_{\text{bn}}) - \overline{\text{ACP}}]$$

and $\frac{\partial G}{\partial \text{BNPC}} = \text{BNGR} - \lambda \psi_{\text{bn}} = 0$

thus $\text{BNGR} = \lambda \psi_{\text{bn}}$

where $\lambda = \text{Lagrangian multiplier}$

Since a production function has not been formally specified, an analytical solution to the above optimization problem cannot be derived explicitly. One can only infer that optimal factor use, in this case cocoa bean grindings, depends on the marginal physical product of cocoa beans. The value of the marginal products, assuming that producing butter fat is the main reason for the grinding activity, is equal to the price of cocoa butter multiplied by its fat content. It is more convenient to express such returns to processing for butterfat in terms of "processing margins" (FTMG) and measure it as

$$\text{FTMG} = \frac{\text{cocoa butter} * \text{fat content} - \text{bean price}}{\text{bean price}}$$

The other important determinant of the level of grindings is the bean input cost. Grindings in period t are therefore a function of the demand for grindings derived from the rate of use in period t of liquor, butter and powder as well as two other factors: one is the cost of the input (cocoa beans), while the other is the net processing margin. The following equation is suggested:

$$\text{BNGR} = f (\text{BNGRDM}, \text{FTMG}, \text{BNPCD})$$

with

$$\text{BNGRDM} = \frac{\text{LQDM}}{0.8} + 0.5 \times \left[\frac{\text{BTDM}}{0.47} + \frac{\text{PWDM}}{0.53} \right]$$

where BNGRDM = derived demand for grindings

BNPCD = deflated bean price

62. We encountered various difficulties in attempting to estimate the above equations. The following discussion reports the major data deficiencies and proposes alternative methods for determining the consumption and output of cocoa products when these data are not observable.

Data Deficiencies

63. The principal areas in which data are lacking is in the consumption of cocoa products--the direct use of liquor, and cocoa butter and cocoa powder demand. Several options to overcome these data deficiencies suggest themselves. In the first place it may be assumed that demand is observationally equivalent to production over the sample period. Given the vertically-integrated nature of cocoa products and downstream manufacturing, this assumption is not entirely unrealistic. For example, liquor (which imparts the flavor in chocolate) can be produced to the level required for chocolate and cocoa butter and presscake manufacture. There has, however, been an erosion of the integrated production structure as the bean-producing

countries have become important suppliers of cocoa products. Industry in certain consuming countries, the Netherlands and the Federal Republic of Germany, for example, has also been an important non-captive supplier of cocoa butter and powder. Also with merchant supplies and markets for cocoa products available it is evident that firms hold stocks of cocoa products rather than hold all stocks in the form of beans. In consequence, the assumption that cocoa products demands are observationally equal to their production needs modification even though regressions for cocoa products demand as functions of chocolate output (CHPD) and the product's relative price vis-a-vis chocolate wholesale prices show quite good fits.

64. A second approach would be to construct data on the demand for cocoa products directly from recipes for chocolate confectionary and chocolate-flavored products. Such an approach would have to be at an individual country level since recipes and the composition of chocolate confectionary (solid chocolate, filled bars, etc.) vary greatly from country to country. Such an approach would also need to estimate import and export functions for the cocoa products. Since recipe information is not available at the country level, certainly not on a yearly basis, this approach is not feasible. An estimate of the average world recipe for chocolate confectionary and chocolate-flavored products can be obtained simply by regressing world grindings on world chocolate production. Given the increasing importance of supplies of cocoa products from bean-producing countries it might be preferable to estimate this regression over the period before 1973, from which date a large volume of cocoa products started flowing from the origins. By applying the respective product technical conversion factors to grindings, estimates of liquor, butter

and presscake/powder in chocolate confectionary and flavored products' recipes can be derived.

65. Another way to derive estimates of the demand for cocoa products is to maintain the assumption of equivalence between the demand for cocoa products and production in the chocolate-producing countries while, at the same time, interpreting imports of cocoa products from the bean-producing countries as a component of the demand for cocoa products. The difference between the output of cocoa products in the bean-producing countries and their exports would be taken as the change in stocks.

66. With either of the latter two approaches, it is evident that the estimated demand will be subject to a certain degree of measurement error. In the model specified above the demand for cocoa products enter the world grindings equation as regressand variables; hence, there are unlikely to be untoward major econometric consequences from the measurement errors. Where they enter as regressor variables, it is necessary to employ an instrumental variable estimation procedure in order to obtain consistent estimates.

Estimates of the Demand and Supply of Cocoa Products

67. The methods adopted for estimating the demand and supply data of cocoa products are based on knowledge of the processing technology as well as on the worldwide market behavior of manufacturers.

68. **Consumption:** Data on the consumption of cocoa butter, powder and liquor by individual countries are not available. Thus intermediate consumption of cocoa products is calculated at the world level in order to avoid compounding possible measurement errors. It is calculated simply as the sum of bean equivalent production (i.e. grindings times the product yield) by the industrial countries and the net exports by the bean-producing countries. This method of construction is consistent with the last of the three approaches described earlier and seems the most suitable given currently available information. Thus the total intermediate use of product can be specified as follows.

$$WDDM_i = y_i * INDGRD + DEVNX_i$$

where

$WDDM_i$ = world consumption of cocoa product i

y_i = yield of cocoa product i per unit of cocoa bean ground

$DEVNX_i$ = net exports of cocoa products by bean-producing countries

$INDGRD$ = grindings of cocoa beans by industrial countries

69. **Output:** The construction of output data was necessary because the only published information (by the United Nations in Industrial Statistics) is sketchy and is lacking for important cocoa consuming countries such as the United States and some Eastern European countries including the USSR. Global output of cocoa product is thus expressed as a technical ratio of world grindings.

70. Consumption and production of each of the cocoa products was estimated in the following ways.

(i) Cocoa Liquor Production

71. There are three methods of approximating the quantity of cocoa liquor directly used in chocolate confectionaries, based on:

- recipe information
- sampling, and
- econometric construction

72. **Recipe information:** Given the confidentiality surrounding such information, manufacturers would not provide exact figures. However, they confirmed that our estimates were within a reasonable range. The range for the amount of cocoa liquor directly used in chocolate is 8-14%, depending on the manufacturer and the confectionary. We also noticed significant differences across countries. Because of these differences, we considered obtaining the average use from a "global" recipe of world chocolate production via the following alternatives.

73. **Chemical analysis of chocolate confectionaries:** The most direct way of obtaining the quantity of cocoa liquor present in a candy bar is by chemical analysis. In 1986 technicians of the magazine Consumer Reports conducted such experiments on 58 brands of chocolate bars (see Table 4). From their cross-sectional analysis we estimated the unweighted average liquor content to be 11%.

TABLE 4: COCOA MASS OF LEADING CHOCOLATE
BAR BRANDS SOLD ON THE US MARKET

	SAMPLE SIZE	COCOA MASS PERCENTAGE	
		RANGE	AVERAGE
DARK CHOCOLATES	17	13-25	17
MILK CHOCOLATES	16	2-16	6
WHITE CHOCOLATE	3	0	0
CHOCOLATE BAR PLUS	22	2-21	9
AVERAGE BAR	58	2-25	11

SOURCE: CONSUMER REPORTS, NOVEMBER 1986, PP. 700-1.

74. According to this study the cocoa content of chocolate confectionaries sold on the US market varies from zero for white chocolate ^{1/} to 25% for some dark chocolate brands.

75. For an estimate of worldwide liquor use, the unweighted US average would be a gross approximation because of the differences across countries in consumer preferences by categories of chocolates. Europeans, especially the British, tend to have a preference for milk chocolate over dark chocolate. Also, European consumers seem to prefer chocolates with less filling than their US counterparts.

^{1/} These products are still called "chocolate" because they contain cocoa butterfat to provide the melting property. The chemical analysis found at most only some traces of cocoa flavor.

76. **Econometric estimation:** A simple approach to estimating the quantity of liquor directly used in confectionaries at a world level is a regression of world grindings on chocolate output. This statistical relation can be viewed as a reduced-form, input demand equation derived from the production of confectionaries.

Thus

$$\text{BNGR} = \alpha \text{CHPD} + \text{constant}$$

multiplying throughout by 0.8 yields

$$0.8 * \text{BNGR} = 0.8 (\alpha \text{CHPD} + C) ; \text{ where } C \text{ is constant.}$$

77. The left hand side of the above relationship is equivalent to the total amount of liquor produced. It can be further decomposed as the sum of liquor used directly (LQUS) and the liquor used for pressing (LQAV). Thus

$$\text{LQUS} = (0.8 * \alpha) \text{CHPD} - \text{LQAV} + C$$

From the above, the coefficient (0.8α) of CHPD can be interpreted as an indicator of the average recipe for liquor used directly. In the estimated regression, α was equal to 0.13. Thus 0.8α , or about 10%, can be considered the average liquor content of chocolate confectionaries. This figure is close to the 11% liquor content resulting from the chemical analysis reported above.

78. The implied recipe information from the regression analysis was selected as the basis for generating the butter and powder production estimates.

ii. Cocoa Butter and Powder Production and Consumption Data

79. Subtracting the amount of liquor used directly for chocolate confectionary from total liquor produced (LQPD) leaves the balance available to be pressed for cocoa butter and powder (LQAV). Assuming an average fat content of powder of 15% and that powder (PWPD) and butter (BTPD) are jointly produced in fixed proportions, the amount of the two products can be estimated as follows:

$$\text{BTPD} = 0.47 * \text{LQAV}$$

$$\text{PWPD} = 0.53 * \text{LQAV}$$

The demand for cocoa butter can be estimated by adding the supplies from bean-producing countries to the cocoa butter yielded by grindings in industrial countries. Intra-industrial country trade in cocoa butter can be ignored.

$$\text{BTDM} = 0.37 * \text{INCBNGR} + \text{BPCBTNC}$$

where

$$\text{BTDM} = \text{World use of cocoa butter}$$

$$\text{INCBNGR} = \text{Grindings in industrial countries}$$

$$\text{BPCBTNX} = \text{Net exports of cocoa butter by bean-producing countries}$$

80. Powder use is derived as a residual in the sense that unused cocoa fat (i.e. available liquor less pressed cocoa butter) is converted into powder of various fat content. If cocoa powder consumption is in the same proportion as implied by the FAO conversion ratios in which high-fat (22-24%) and low-fat (10-12%) powders are produced, i.e. one-third and two-thirds respectively, the average fat content is 15%. Powder use can therefore be estimated as.

$$PWDM = [0.55 * \text{liquor available less butter produced}] / 0.15$$

81. For the purpose of determining powder prices, it is preferable to do the analysis in terms of unused cocoa fat rather than arbitrarily converting it to cocoa powder use by applying an average fat content factor. The former is the procedure used in this study. If interest is focused on the end uses for cocoa powder, clearly a more refined approach using recipes, alternative flavorings and their prices needs to be incorporated in order to estimate powder use by type of powder.

CHAPTER V. PRODUCT PRICE FORMATION: THEORETICAL DISCUSSION
AND EQUATION SPECIFICATION

Review of Literature

82. The aim of the paper is to model the determination of cocoa product prices and derive a relationship linking bean and product prices. Given the characteristics of the cocoa bean and its derived products, a logical source of inspiration for modeling such price behavior seems to be in the joint production literature.

83. The theory of joint production is not new. However, it was only relatively recently that it has been carried beyond the textbook examples of wool and mutton, iron and steel, or beef and hides etc. ^{1/} Laitinen has extended the theory of the traditional, homogeneous, single good firm to the multiproduct case. Pasinetti (1980) has also applied the joint production theory to analyze the bivalent nature (in terms of stocks and flows) of capital goods. The general problem these studies try to solve is that of an optimal transformation of the joint outputs in a multi-input, multi-product framework. Dynamic extensions of this methodology have been tried by some authors to solve the temporal optimization of a multiproduct firm (Laitinen).

^{1/} For a comprehensive review of joint production studies, see Laitinen (1980) pp. 2-12.

84. The problem of interest in the current project is purposely kept more in the vein of the classical examples in the sense that the joint products are produced in nearly fixed proportions; although by changing technology the proportions can be changed. Therefore, there is no form of arbitrage. In the cocoa products' economy, several technologies are available for converting beans into butter and powder--with each yielding relatively fixed proportions. The problem of the converter is to choose among these technologies.

85. If cocoa butter and hence cocoa powder are fixed-yield, joint products of cocoa beans (the beans contain a fixed fat content and technologies succeed to a varying extent in recovering the fat content), the linkage between product and bean prices could be specified in the following theoretical relationships:

$$(i) \quad BNPC = w_1 BTPC + w_2 PWPC + w_3 LQPC$$

$$(ii) \quad E_{bn} = \frac{w_1 BTPC + w_2 PWPC + w_3 LQPC}{\frac{1}{E_{bt}} (w_1 BTPC) + \frac{1}{E_{pw}} (w_2 PWPC) + \frac{1}{E_{1q}} (w_3 LQPC)}$$

86. Relationship (i) simply states that the price of cocoa beans is the weighted sum of the prices of the twin products--with the weights being the yields per unit of beans. This relation could be imposed as a consistency constraint that prices must satisfy in equilibrium and when products are valued solely on the basis of their fat content. Because liquor is produced for direct use there is no reason for (i) to hold and in fact it rarely holds in practice.

87. Relationship (ii) is derived from (i) but imposes an even more strict restriction on the elasticities. 1/ Relation (ii) holds if, as should be the case when joint products are obtained in fixed proportion, the price demand elasticity of beans is the weighted harmonic average of the price elasticities of butter and powder.

88. Another interesting relationship to observe, if margin equations could be estimated, is the one suggested by Brandow (1961). He assumes that if margins are a linear function of output,

$$M = k + aAx \quad \text{where } 0 \leq a < 1 \\ \text{and } k \geq 0$$

where M = margins, Ax = output, and k = a constant.

the raw (bean) demand elasticity in our case becomes:

$$(iii) \quad E_{bn} = E_r \left(1 - \frac{k}{(1-a) Ax} \right),$$

where E_r is the retail level elasticity (for cocoa-based end products).

Relation (iii) could be more practical and relevant to our purpose since cocoa butter and cocoa powder have two separated end use markets (chocolate for butter and bakery; dairy or drinks for powder).

1/ Mathematical derivation of relation (ii) for a two joint-product case is presented in Annex F.

89. So relation (iii) suggests a linkage of the cocoa model blocks through a simple margin equation.

90. Although neither of the above relationships are tested in the analysis, they could be exploited in further model developments. However, as will be demonstrated, omission of such a relationship does not invalidate the analysis as long as observed and predicted price ratios fall within certain boundaries and the analysis is conducted within such limits.

Formulations of Product Price Equations

91. The model of price formation developed in the current study follows from the assumption that each product market clears in the traditional manner exposted by Arrow (1959). Thus, a price adjustment equation that equates demand to supply or that nullifies the excess demand relation is specified for each product.

$$S = S(P, \alpha)$$

$$D = D(P, \beta)$$

$$\frac{dp}{dt} = H (q_d - q_s) = H [D(P, \alpha) - S (p)]$$

or

$$\Delta P = f (\text{excess demand})$$

92. In the usual treatment, equilibrium prices are found by solving for P and assuming that supply equates to demand without any explicit explanation of

the trial-and-error processes that stabilize the above system. The price equations specified in this study attempt to describe explicitly the adjustment behavior of cocoa bean processors in their grinding and pressing operations. This calls for an alteration of the above adjustment equation as follows:

$$\Delta P = f(\text{excess demand, processing margins})$$

There are several reasons for adding the second explanatory variable, processing margins. One reason is the absence of a terminal market for these products similar to those of New York or London for cocoa beans. In addition, because liquor, butter and powder are intermediate products, their pricing will reflect a cost-plus element.

93. The specific formulation of each product price equation will be discussed in subsequent sections. A general form for such equations is

$$\Delta PP_{it} = \lambda_i [PP_i^e - PP_{(i-1)}] + XDP ; \quad \forall i > 1$$

where

- PP = product price
- PP^e = "warranted" price
- XDP = excess demand
- λ = adjustment parameter
- i = 1, 2, 3 and 4 for respectively bean, liquor, butter and powder.

94. Further definitions of "processing margins" and "warranted" price are in order. In the current context, processing margins should not be interpreted as mark-up, the component of price above the average variable cost added to cover overhead costs and profits. Instead, margins are interpreted here, following a less conventional definition (Tomek and Robinson, 1981, p.122), as the price of specific marketing services that satisfy time, space and form utilities. The current study is more concerned with form utility, in that product prices are hypothesized to include the cost of the service of transforming cocoa beans into various products. The supply and demand for such a service depends on input costs of which the cocoa bean price is an important component. 1/

95. The "warranted" price is the value of the product such that the market is form efficient--meaning that the raw and derived products' prices are separated by the processing costs. In the absence of processing cost data, inference about form efficiency can be made from the wedge between the actual price of the derived product and the raw product price times the technical conversion factor. It is this latter entity, (raw product price times the technical conversion factor) that is called the "warranted" price. Its inclusion in the price adjustment behavior model is consistent with optimality conditions of the multiproduct firm whereby product price ratios should equal the ratios of the rates of product transformation.

1/ Further discussion of pricing of joint-services with alternative product forms is found in Bressler and King (1970, Chapter 9).

96. Turning back to the price adjustment equation, the parameter λ accounts for the technical lag in changing processing technologies to adjust prices to their "warranted" level with respect to the bean price. The term in the square brackets is a proxy for the expected profitability of processing; whereas the last term is a measure of excess demand for an individual product.

97. To summarize, the λ_i and PP_i^e of individual product price equations of the general form written above are developed as

$$\lambda_i = f(\text{STGRR}); \quad i > 2$$

where STGRR = stocks to grindings ratio,

and

$$\text{for liquor: } PP_2^e = f(\text{BNPC})$$

$$\text{for butter: } PP_3^e = f(PP_2, PP_3); \text{ and}$$

$$\text{for powder: } PP_4^e = f(PP_3, PP_2)$$

98. The adjustment parameter for both butter and powder can be made a function of the stocks-grindings ratio on the grounds that adjustment of prices for products to the level of processing required to satisfy the

expected demand for grindings will vary with the amount of beans available. ^{1/}
The estimation of the price equations using λ specified as stated above works well for butter but not for powder. This result is consistent with the fact that butter has for a long time been considered the main product in the processing of cocoa beans. The individual product price equations estimated are discussed below.

Liquor Price Equation

99. The liquor price equation takes the following formulation:

$$\Delta LQPC = \lambda_1 [\alpha_1 BNPC - LQPC_{-1}] + LQXDM$$

where $LQXDM$ = excess demand for liquor

^{1/} See Abramovitz (1959) and also Nerlove (1972) for a general theoretical discussion of price formation and adjustment costs. Peel and Walter (1983) tested for the case of the labor market, the hypothesis that the speed of adjustment of the actual to the desired level of employment is partly determined by the size of the excess supply of labor. By analogy, it is claimed here that the availability of beans will influence λ , which in turn, with product in excess demand, will affect the price adjustment at period t , (ΔPP_{it}) .

100. A rigorous specification of the excess demand for liquor requires logically to subtract grindings (the supply element) from the bean-equivalent liquor demand. However, since the demand for liquor is not observable, it seems reasonable to construct the "demand component" of an hypothetical excess demand for liquor from chocolate production. Thus a proxy for liquor in excess demand is formulated as follows.

$$LQXDM = \alpha + \beta CHPD + \gamma BNCR + \eta STGR$$

101. The CHPD term measures the demand aspect. Other demand effects could be captured by including chocolate-flavored goods production; but this additional variable would introduce multicollinearity with CHPD. The constant term α is included to pick up any residual demand effect.

102. The "supply component" of the hypothetical excess demand is proxied by grindings, BNCR, and the stocks-to-grindings ratio. The latter variable is added to compensate for the absence of the (unobservable) liquor stocks. In addition, STGR will reflect the effect of potential liquor supply since the size of stocks naturally influences the availability of beans for liquor processing and hence excess demand for liquor. The term STGR is developed as the ratio of lagged stocks to current grindings as cocoa liquor is pressed from the current crop and from existing stocks.

Putting all the pieces together, the liquor price equation becomes

$$\Delta LQPC = \lambda_1 [\alpha_1 BNPC - LQPC_{-1}] + \alpha + \beta CHPD + \delta BNGR + \eta STGR$$

The priors on the coefficients are as follows:

$$\begin{aligned} 0 < \lambda < 1 \\ \alpha > 0 \\ \beta > 0 \\ \delta < 0 \\ \eta < 0 \end{aligned}$$

The prior on the adjustment parameter of cocoa liquor indicates that the adjustment of the actual liquor price, $LQPC$, to the "warranted" liquor price, $LQPC^e$, is not instantaneous. The "warranted" price is a function of the cocoa bean price. Since the latter constitutes a cost to the cocoa liquor presser, α_1 should be positive. The coefficient β for the "demand side component" $CHPD$ of the excess demand of cocoa liquor should also be positive. The coefficients δ and η should be negative since they relate to the supply side components, grindings and bean stocks, of the excess demand for liquor.

Cocoa Butter Price Equation

As a first approach we can specify

$$\Delta BTPC = \lambda_b [BTPC^e - BTPC_{-1}]$$

On the basis of the common view among processors that butter is valued at the fat content of the cocoa bean, this equation can be respecified as

$$\Delta BTPC = \lambda_b [\alpha_b BNPC - BTPC_{-1}]$$

103. The above formulation implies that α_b should be close to the butter conversion rate 1.33. Unlike the liquor rate, this conversion factor is derived from an arbitrary assumption regarding the fat composition of the presscake/powder (see previous chapter). Thus it is not expected that α_b should turn out to be exactly 1.33.

104. A reformulation of the equilibrium butter price, $BTPC^e$ or $\alpha_b BNPC$, is necessary given the weakness of the adopted bean price proxy in conveying information about quality premia for butter from different sources. The International Cocoa Organization price used as the indicator for cocoa prices is a futures price that has not been adjusted for quality differentials. Furthermore, taking into account the breakdown of vertical integration in the cocoa industry it seems appropriate to determine $BTPC^e$ in terms of other product prices rather than solely in terms of bean prices. Thus,

$$\text{BTPC}^e = f(\text{LQPC}, \text{PWPC})$$

The liquor price can be considered predetermined to the presser. Similarly, where butter is the main product, the powder price is the by-product credit. The two price variables are sufficient to fix the value that the processor would give to butter since they determine the processors' operating margin.

105. Observed prices may deviate from the "warranted" price for several reasons. For instance, actual prices can differ from the "warranted" level in the case of excess demand for butter. Usually, such excess demand exists when there is a shift in recipe in favor of milk chocolate and away from dark chocolate. To capture this tendency, the variable butter use/liquor use ratio was chosen to proxy the excess demand for butter. The more conventional measure of excess demand for butter is butter use less butter supply. Butter supply can be constructed as defined earlier as the product of the liquor available and the fat content plus expeller butter. However, since historical data for expeller butter production do not exist, an equation for that component of butter supply could not be estimated. Butter use is also unobservable and has to be proxied by CHPD. Against this background the measure proposed, ratio of butter to liquor use, appears as the least problematic proxy.

106. So the equation to be estimated for the butter price becomes

$$\Delta BTPC = \lambda_b (\alpha_b LQPC + \alpha'_b PWPC - BTPC_{-1}) + a_1 + a_2 BTLQUSR$$

with $\lambda_b = f(STGR)$

where $BTLQUSR =$ ratio of butter use to liquor use

The priors for the cocoa butter price equation are:

$$0 < \lambda_b < 1$$

$$\alpha_b > 0$$

$$\alpha'_b < 0$$

$$a_2 > 0$$

The rationale for the expected signs on λ_b and α_b is the same as that for λ_1 and α_1 in the case of the cocoa liquor price equation. The coefficient α'_b for the cocoa powder price PWPC should be negative to reflect the joint product nature of cocoa powder and cocoa butter. When the cocoa butter price is high, the cocoa powder price is low as a result of either a second-round effect of an increasing supply of cocoa butter (and thus cocoa powder), or as a result of the willingness of manufacturers to accept lower bids on cocoa powder. The coefficient a_2 on BTLQUSR, representing an intermediate demand, should be positive.

Powder Price Equation

107. Following the development of the butter price equation, the powder price equation can be respecified as

$$\Delta PWPC = \lambda_p [PWPC^e - PWPC_{-1}] + PWXDM$$

where $PWXDM$ = excess demand for cocoa powder

108. A simple way of formulating $PWPC^e$ is to relate it to the input price (liquor price) and the main product price (butter price). In the case of the powder price we do not make the a priori claim that the powder-bean ratio should tend to the technical conversion ratio. As for the excess demand variable, the demand and supply for powder can be represented respectively by CFLPD (cocoa flavored output or the food production index) and the unused fat (i.e., the difference between the liquor available and the cocoa butter used). If one makes the arbitrary assumption of 12 to 15% fat content in cocoa powder, the unused fat variable is measured as liquor available times 0.53.

The powder equation then becomes

$$\Delta PWPC = \lambda_p [PWPC^e - PWPC_{-1}] + C_0 + C_1 FTBI + C_2 UNUSFT$$

or

$$\Delta PWPC = \lambda_p [\alpha_{p1} LQPC + \alpha_{p2} BTPC - PWPC_{-1}] + C_0 + C_1 FTBI + C_2 UNUSFT$$

where FTBI = food, tobacco and beverages index

UNUSFT = unused cocoa fat.

The priors for the cocoa powder price are:

$$0 < \lambda_p < 1$$

$$\alpha_{p1} > 0$$

$$\alpha_{p2} < 0$$

$$c_1 > 0$$

$$c_2 < 0$$

The rationale for the expected signs on λ_p , α_{p1} and α_{p2} follow the same logic as that in the case of the price of cocoa butter. The demand side element, the proxy for biscuit production (FTBI) should be positively related to the price of cocoa powder; whereas the supply side element, the unused cocoa fat, relates negatively to it.

VI. ESTIMATION OF COCOA PRODUCT DEMAND AND PRICE EQUATIONS

Grindings Equation

109. To review the previous discussion, grindings are specified as a function of the expected demand for grindings, the expected margins recovered from the grinding operation and the deflated bean price. The estimated consumption data allows the computation of a demand-for-grindings variable which is measured as the bean equivalent sum of the liquor, butter and solids use. This demand for grindings is satisfied from current grindings and from changes in the inventories of cocoa products. The demand for grindings is therefore a predetermined variable to the grindings decision and, hence, one can specify a grindings equation as the solution to the problem of minimizing costs of grindings given a predetermined demand and the bean and products prices.

110. By modeling grindings in this manner it is possible to capture the effect on grindings of the relative demands of the three cocoa products and the relative price structure of the products vis a vis each other and with respect to the cocoa bean price.

111. The estimated grindings equation is reproduced below 1/

$$\begin{aligned} \ln \text{BNGR} &= 0.2561 + 0.8817 \ln \text{BNGRDM} + 0.0327 \ln \text{FTMG} \\ &\quad (1.27) \quad (15.62) \quad (2.27) \\ &+ 0.0924 \ln \text{BNGR}(-1) - 0.0177 \ln \text{BNPCD} \\ &\quad (1.68) \quad (-4.13) \\ \bar{R}^2 &= 0.99 \quad \text{DW} = 1.64 \end{aligned}$$

112. In this specification the variable FTMG can be viewed as the difference between the liquor/bean price ratio and the returns to liquor pressing (butter and powder prices) relative to the liquor price. Grindings are positively related to this difference. When this difference is low, i.e., when FTMG takes on a low value or if returns to pressing are high relative to the liquor price, grindings tend to decline. Such a decline would work its way through to an increase in the liquor price and hence into FTMG in the subsequent period. The experience of 1980-83 is a good illustration of this relationship when the annual increase in grindings was limited because the liquor/bean price ratio remained fairly stable while returns from butter and presscake were increasing.

113. The short- and long-run price elasticities of grindings with respect to changes in the cocoa bean price (BNPC) are derived from computing the following expression:

1/ Most variables have been already defined in previous sections; but to enhance the reading of the subsequent equations, the variable descriptions are repeated in the table at the end of this chapter.

$$\frac{d}{d \log \text{BNPC}} \ln \text{BNGR} = \frac{d}{d \log \text{BNPC}} [-8817 \ln \text{BNGRDM} + .0327 \ln \text{FTMG} \\ -0.0177 \ln \text{BNPCD}]$$

114. Given the combination of equations in log and arithmetic forms, we can only say something about elasticities at sample means. (At sample means the ratio BNPC/BNGR is close to unity, which facilitates the computations.) Thus, the short- and long-run price elasticities of world cocoa grindings at sample means are respectively -0.4 and -0.45. Therefore, by including the relationships between cocoa products and grindings, the estimated short-run price elasticity turns out to be much larger than the previously estimated elasticities of around -0.25 when products were not included in the specification. In specifications which have not included cocoa products, the long-run elasticity estimates have been found to be fairly close to that estimated above, confirming indirectly the above approach.

115. A larger elasticity estimate was expected since beans are ground not only in response to signals from the end-product market but also in response to changes in the relative profitability of using cocoa fat instead of other vegetable fats. In turn, cocoa product demand (a component of total foodstuffs) is essentially driven by income and population growth. At the world level, income and price elasticities of cocoa product demand are bound to be small as is expected of most food items. However, the fact that other vegetable fats can substitute for cocoa fats increases the price elasticity.

Cocoa Products Demand

116. Estimation of the cocoa products demand relationships presented earlier gave the following results.

Cocoa Butter Demand:

$$\begin{aligned} \text{BTDM} &= 140.5367 - 22.6270 \text{ BTCHPCR} - 41.9642 \text{ BNCHPCR} \\ &\quad (4.3999) \quad (-1.1976) \quad (-2.6045) \\ &+ 0.7544 \text{ BTDM}(-1) - 0.6256 \text{ BTFTPCR} \\ &\quad (7.7750) \quad (-1.0781) \\ &+ 0.0240 \text{ CHPD} \\ &\quad (1.7897) \\ \bar{R}^2 & 0.892 \quad \text{s.e.e.} = 12.769 \quad \text{DW } 2.43 \end{aligned}$$

117. The chocolate-flavored output variable was dropped for the reasons stated earlier.

118. All coefficients have the expected signs although some are not significantly different from zero at the 95% confidence level. The t-value for the coefficient of BTCHPCR is low because that variable is certainly correlated with BNCHPCR.

119. Inclusion of the variable BTFTPCR is intended to account for the impact of substitute non-cocoa fat (NCF) use in chocolate confectionary. The results suggest that such an effect has not been significant--which is in line with reality in the United States where NCFs are prohibited and in the EEC where their use is strictly regulated or limited, as in the United Kingdom.

Cocoa Powder Demand:

$$\text{PWDM} = 191.0189 - 40.2711 \text{ PWCHPCR} + 0.4468 \text{ PWDM}(-1)$$

(4.6576) (-4.2804) (3.4646)

$$+ 28.6580 \text{ BSCHPDR}$$

(2.0125)

$$\bar{R}^2 = 0.745 \qquad \text{s.e.e.} = 16.043 \qquad \text{DW } 1.63$$

120. The price elasticity implied by the above input demand equation is -0.06. Such a low elasticity is consistent with our earlier explanation of the quadripling of cocoa powder prices during the period 1976-1978 when there was a shortage of cocoa beans. It is also consistent with the facts that, as stated earlier, cocoa powder has virtually no substitutes in the chocolate flavoring industry and that expenditures on cocoa powder represent a relatively small proportion of the input requirement of end product manufacturers.

121. The equation furthermore shows that the level of chocolate-flavor production activity influences positively the quantity of intermediate consumption of cocoa powder.

Cocoa Products Price Equations

122. The prices of cocoa products are hypothesized to adjust to their equilibrium values which are determined by either the technical conversion rates (from bean to liquor) or, where there are joint products, by technical conversion ratios modified to take account of relative demands for the joint products. Effectively, product prices adjust to eliminate excess processing

margins in their production. A second component in the price equations is the degree of excess demand for the cocoa product in question.

The estimated price equation for liquor is as follows:

$$\begin{aligned} \Delta \text{LQPC} = & 1422.97 + 0.924 \text{BNPC} - 0.841 \text{LQPC}_{-1} \\ & (2.57) \quad (14.86) \quad (12.7) \\ & -1706.4 \text{STGR} + 0.106 \text{CHPD} \\ & (3.6) \quad (0.76) \\ & -0.72 \text{BNGR} \\ & (1.6) \\ \bar{R}^2 = & 0.95 \quad \text{s.e.e.} = 132.17 \quad \text{DW} = 2.23 \end{aligned}$$

The estimated price equation for butter is

$$\begin{aligned} \Delta \text{BTPC} = & -2558.17 + 5.09 * \text{STGR} * \text{LQPC} \\ & (1.3) \quad (5.74) \\ & -1.651 * \text{STGR} * \text{PWPC} \\ & (2.77) \\ & -1.681 * \text{STGR} * \text{BTPC}_{-1} \\ & (5.4) \\ & + 1394.7 * \text{BTLQUSR} \\ & (1.66) \\ \bar{R}^2 = & 0.80 \quad \text{s.e.e.} = 294.33 \quad \text{DW} = 1.806. \end{aligned}$$

Finally, the estimated equation for powder prices is

$$\begin{aligned} \Delta \text{PWPC} = & 1271.67 + 1.405 * \text{LQPC} - 0.766 * \text{BTPC} \\ & (2.67) \quad (15.65) \quad (7.51) \\ & 0.6714 * \text{PWPC}_{-1} + 7.97 * \text{FBTI} \\ & (11.33) \quad (1.002) \\ & -26.443 * \text{UNUSFT} \\ & (1.99) \\ \bar{R}^2 = & 0.94 \quad \text{s.e.e.} = 177.5 \quad \text{DW} = 2.05 \end{aligned}$$

123. Considerable insight is cast on the estimated equations by rewriting them in accordance with the theoretical partial adjustment model suggested above and substituting the empirical counterparts for equilibrium prices. This reformulation yields:

$$\Delta \text{ LQPC} = 0.84 [1.1 \text{ BNPC} - \text{LQPC}_{-1}] - 1706.0 \text{ STGR} \\ + 0.106 [\text{CHPD} + 14229 - 7.16 \text{ BNGR}]$$

$$\Delta \text{ BTPC} = 1.68 * \text{STGR} * [3.03 \text{ LQPC} - \text{PWPC} - \text{BTPC}_{-1} + 1394 \text{ BTLQUSR} - 1.83]$$

$$\Delta \text{ PWPC} = 0.67[2.09 \text{ LQPC} - 0.14 \text{ BTPC} - \text{PWPC}_{-1}] \\ + 7.97 [\text{FBTI} + 159.6 - 3.3 \text{ UNUSFT}]$$

The equations can now be interpreted in terms of the reasoning outlined earlier. The first term in the brackets on the right hand side of each equation represents a measure of the expected margins in the processing of each product, or, put another way, the difference between the "warranted" price and the lagged price. In the case of butter prices, the parameter expressing adjustment of the butter prices to the expected margin is also a function of the ratio of stocks to lagged grindings.

124. Estimation of the cocoa product price model indicates that the adjustment of prices to their warranted level is not instantaneous and that, in fact, product prices are seldom a fixed (technical conversion factor) multiple of cocoa bean prices as implied by those models which omit cocoa products sector. For example, the cocoa liquor price adjustment parameter is less than unity (0.84) and the liquors price is related to the bean price by a factor of 1.11, not by the technical conversion factor of 1.25. This finding has an important implication as the cocoa product-cocoa bean price ratio is

often used as a guide in assessing the costs-benefits of processing cocoa products versus exporting raw cocoa beans, as well as in the choice of the processing technology. The speed of the adjustment of cocoa butter prices to their warranted level varies positively with the size of cocoa bean stocks available for pressing. The average adjustment parameter is 0.6. But when cocoa beans were relatively scarce as in 1977, it decreased to 0.32-- suggesting a slow adjustment of cocoa butter prices to their warranted level. The fastest adjustment occurred in 1965, which is also the year the ratio of cocoa bean stocks to grindings, reached its highest level. The adjustment coefficient was close to 1.0 and the bean stocks to grinding ratio was 0.79.

125. The second term in square brackets on the right-hand side of each equation is a measure of excess demand for the product. In the liquor price equation, demand for liquor is represented as a function of chocolate output while liquor supply is represented by grindings. Similarly, in the powder equation, demand for powder is a function of the food output index while supply is the quantity of unused fat. In the butter equation the demand for butter is directly measured as butter use. The supply of butter is difficult to measure directly since it depends on the quantity of liquor produced less liquor directly used in chocolate manufacture. The supply of butter is thus estimated as a constant multiple of liquor use.

126. A summary of the results of the key estimated equations of the cocoa product market model is presented in Annex E.

Equilibrium Product Prices

127. The equilibrium or warranted prices can be derived from the simultaneous equation system set out below.

$$LQPC^e = 1.1 \text{ BNPC} \quad (1)$$

$$BTPC^e = 3.03 \text{ LQPC} - 0.98 \text{ PWPC} \quad (2)$$

$$PWPC^e = 2.10 \text{ LQPC} - 1.14 \text{ BTPC} \quad (3)$$

The system represents three equations and four unknowns and assumes that $LQ = QPC^e$, $BTP = TPC^e$, and $PWPC = PWPC^e$. Otherwise, there would be seven unknowns with three equations. However, there are three equations that can be added to the above system.

$$LQPC = LQPC^e + \Gamma_1 \quad (4)$$

$$BTPC = BTPC^e + \Gamma_2 \quad (5)$$

$$PWPC = PWPC^e + \Gamma_3 \quad (6)$$

here Γ_i = the respective excess demand variables.

By substitution, the consolidated system of six equations and seven unknowns can be reduced to the following

$$LQPC^e = 1.1 \text{ BNPC} \quad (1)$$

$$BTPC^e = 3.33 \text{ BNPC} + 3.03 \Gamma_1 - 0.98 \text{PWPC}^e - 0.98 \Gamma_3 \quad (2)$$

$$\text{PWPC}^e = 2.31 \text{ BNPC} + 2.10 \Gamma_1 - 1.14 \text{BTPC}^e - 1.14 \Gamma_2 \quad (3)$$

To solve the system, the simplest approach is to set $LQPC=LQPC^e$ and then set alternatively $\text{PWPC} = \text{PWPC}^e$ and $\text{BTPC} = \text{BTPC}^e$. Since the values of the Γ_i 's are unknown, except for the fact that they depend on bean and product prices, the only way to estimate the relationship between equilibrium butter and powder prices is by assuming that liquor and butter excess demands are nil ($\Gamma_1=\Gamma_2=0$) or that liquor and powder excess demands are nil ($\Gamma_1=\Gamma_3=0$).

128. Using identity (1) and dividing throughout by BNP in the "warranted" price relations (2) and (3) (having assumed that LIQP equals LIQP^e) gives the following

$$\frac{BTPC^e}{BNPC} = 3.03 * 1.1 - 0.98 \frac{\text{PWPC}}{BNPC} \quad (4)$$

$$\frac{\text{PWPC}^e}{BNPC} = 2.1 * 1.1 - 1.14 \frac{BTPC}{BNPC} \quad (5)$$

To solve (4) we set $LQPC=LQPC^e$ and $PWPC = PWPC^e$; this gives (6) below, which represents the line which is the locus of points where $BTPC$ is different from $BTPC^e$. Similarly, to solve (5), we set $LIQP = LIQP^e$ and $BUTP = BUTP^e$; which yields the line represented by equation (7) such that $POWP \neq POWP^e$. That is

$$\frac{BTPC^e}{BNPC} = 3.33 - 0.98 \frac{PWPC^e}{BNPC} \quad (6)$$

if $BTPC \neq BTPC^e$ or $\Gamma_1 = \Gamma_2 = 0$

and

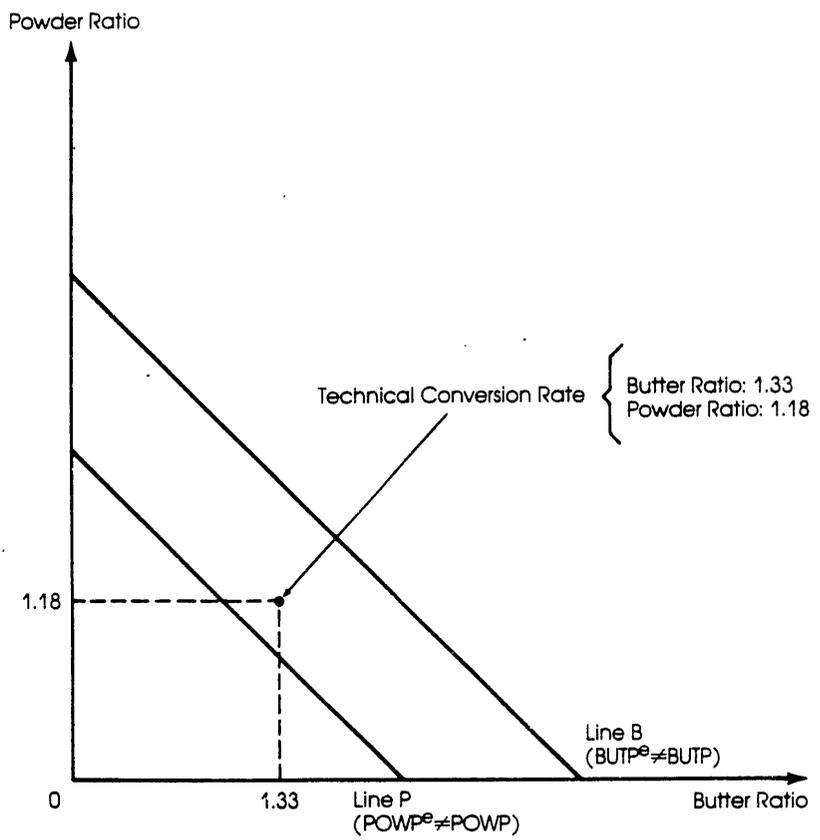
$$\frac{PWPC^e}{BNPC} = 2.31 - 1.14 \frac{BTPC^e}{BNPC} \quad (7)$$

if $PWPC \neq PWPC^e$ or $\Gamma_1 = \Gamma_3 = 0$

Clearly, we need a further equation linking the product prices to the determination of bean prices. Without this, the framework of analysis does not provide relationships that can be used to obtain conditions when all three product prices are at their "warranted" values so that equations (6) and (7) can be solved only in terms of $BNPC$. However, (6) and (7) can be represented diagrammatically as follows in Figure 4.

129. The two equations indicate the region in the butter/bean price ratio and powder/bean price ratio space in which the butter/bean and powder/bean price ratios lie if processing is to be profitable. A comparison with the observed historical butter/bean and powder/bean price ratios shows that they all lie in this region.

Figure 4: Product Price Ratio Space



130. These relationships provide certain useful rules of thumb for processors. Firstly, it is clear that for a reasonably wide band between the lines B and D in Figure 4, the butter/bean and cake/bean price ratios outside the region are indicative of unprofitable operation in terms of butter recovered, the fat content of the cake, and pressing technology. All the more so when other factor costs such as labor, capital and fuel are added.

131. Secondly, knowledge of the region delineated above and additional information as to whether it is the butter or powder market which is out of equilibrium enables the processor to predict how the price ratios will change. From equations (2) and (3), after eliminating the liquor price, we obtain the following relationship between the "warranted" butter/bean and powder/bean price ratios at any liquor/bean price ratio.

$$\frac{PWPC^e}{BNPC} = \text{constant} - 1.4 \frac{BTPC^e}{BNPC}$$

132. We do not know the value of the constant, but that of the slope (-1.4) provides sufficient information with which to judge the direction and magnitude of change in the powder/bean ratio for a given butter/bean ratio and vice versa.

Implications for Cocoa Processing

133. This study has shown that, apart from bean availability, the important elements in the determination of cocoa processing are (i) the direct use of liquor in chocolate production, (ii) the alternative technologies for

cocoa processing and (iii) the relative demands for cocoa butter and cake/powder. These elements have been exploited in order to estimate a model of cocoa product prices. This model shows that around 1977 there occurred a large increase in the direct use of liquor in chocolate confectionary. A number of structural and conjunctural factors account for this upturn in direct liquor usage. What is important for cocoa processing was that the subsequent few years showed an unprecedented increase in the price of powder/cake relative to bean price and a concomittant fall in the butter ratio. Against this background of very substantial changes in the cocoa products basic price ratio vis-a-vis beans, it is interesting to examine, on the one hand, the question of choice of techniques amongst the alternative ways by which cocoa can be processed, and on the other, to ask whether tendency for the emergence of markets of cocoa products, given the substantial increase in non-captive processing which has occurred with the increase of grinding in the producing countries. The recent decision to open a futures market for cocoa butter quite validates the relevance of this inquiry.

134. On the question of choice of techniques, the most appropriate way to evaluate alternative activities is to set-up an optimization problem. This is done by defining the input/output conditions of alternatives such as pressing,

expelling, solvent extraction (or any linear combinations of two or more of these processes) and by setting the problem of the processor as follows:

$$\max \sum_i \pi_i x_i$$

$$\text{s.t } Ax \leq b$$

$$0 \leq x \leq 1$$

where π_i is the profit per unit for activity x_i . The subscript i refers to a given activity which could be (say) pressing; A is a matrix of constraints summarizing the inputs required by the activities and b a vector of input constraints. The vector of input constraints b is determined not exogenously but by the pattern of demands facing the processor. The processor can decide whether to convert the beans at his disposal directly into butter through expelling and solvent extractions or generate first liquor and then adopt hydraulic pressing. In this way the pattern of demands for cocoa products facing the processor determines the most profitable operation amongst pressing, expelling or any combinations. The above optimization applied to the prevailing prices during the period 1980-83 clearly bears out this conjecture. A forecast of product prices which can be used as inputs to the above optimization model provides evidently a means of aiding the investment decisions as to which type of cocoa processing capacity (hydraulic press, expeller, expeller + solvent extractor) should be installed.

135. As regards to investing in cocoa processing in the producing countries, the existing difficulties in disposing of or competing effectively in the heterogenous market for cocoa powders mean that technologies whose joint product mixes are weighted in favor of producing the more homogenous cocoa butter are largely to be more profitable. The homogeneity of cocoa butter and the increasing non-captive production are also factors that contribute to the preconditions necessary for the eventual emergence of a market for liquor and cocoa butter. To some considerable extent the cash markets for the products exists already where the product ratios rather than absolute product prices are traded.

136. The notations used to define variables in this section are described below in Table 5.

Table 5: VARIABLE NAMES AND DESCRIPTION

Name	Description
BNCHPCR	Cocoa bean to chocolate price ratio
BNGR	Grindings of cocoa beans
BNGRDM	Demand for grinding cocoa beans
BNPC	Price of cocoa beans
BNPCD	Price of cocoa beans deflated by food price index
BSCHPDR	Biscuit to chocolate production ratio
BSPD	Biscuit production
BTBNPCR	Cocoa butter to cocoa bean price ratio
BTCHPDR	Cocoa butter to chocolate production ratio
BTCHPRC	Cocoa butter to chocolate price ratio
BTDM	Intermediate consumption of cocoa butter
BTPD	Production of cocoa butter
BTPC	Price of cocoa butter
BTLQUSR	Ratio of butter use to liquor use
FTMG	Fat margin
BTXD	Excess demand for cocoa butter
LQBNPCR	Cocoa liquor to cocoa bean price ratio
LQCHPCR	Cocoa liquor to chocolate price ratio
LQDM	Intermediate consumption of cocoa liquor
LQPD	Production of cocoa liquor
LQPC	Price of cocoa liquor
LQXD	Excess demand for cocoa liquor
PWBNPCR	Cocoa powder to cocoa bean price ratio
PWCHPCR	Cocoa powder to chocolate price ratio
PWDM	Intermediate consumption of cocoa powder
PWPD	Production of cocoa powder
PWPC	Price of cocoa powder
PWXD	Excess demand for cocoa powder
STGRR	Stocks to grinding ratio
CHPD	Chocolate production
CFLPD	Chocolate flavored product production
EXBTPD	Expeller cocoa butter production
PWGHPCR	Cocoa powder to chocolate price ratio
LQAV	Cocoa liquor available for pressing

VII. CONCLUSION

137. The inclusion of the economics of cocoa processing into the determination of the prices of cocoa liquor, butter and powder is a necessary development in modeling of the cocoa market. Moreover, produce price projections are needed for more accurate estimates of cocoa revenues at a time when cocoa bean producing countries are increasingly engaging in downstream processing. These projections also should help in investment choices of appropriate processing facilities. The fact that cocoa products' quotations are offers and bids by large cocoa bean processors, unlike cocoa bean prices which are determined through futures market transactions, necessitates the inclusion of processing margins in the determination of product prices. Cocoa product prices are not simple constant multiples of the cocoa bean price. It is argued here that the variations of cocoa product prices are influenced by the excess demand and the expected profitability of converting cocoa into the downstream product(s). In the model developed here, cocoa product prices affect the price of cocoa beans through the grindings relationship. Grindings are a function of the derived demand for grindings which is the sum of the bean-equivalent expected demand for the individual cocoa products. Because of the paucity of observed product data, the study generates output and intermediate consumption data on cocoa products from technical and econometric relations. The results seem robust by replicating well the historical behavior of cocoa product prices and cocoa bean grindings. By endogenizing cocoa product market decisions, it is found that the price elasticity of bean

grindings is -0.4 . Previous elasticity estimates from analysis in which the cocoa products' price relationship is not specified are around -0.25 .

138. Cocoa powder and cocoa butter prices were found to vary consistently within an analytical framework of joint-product pricing. With the growing demand for "high-fat" powders, cocoa powder is no longer a residual by-product of cocoa butter that can be disposed of at low prices. This tendency should be accentuated as the demand for powder expands. Current market forces also point to a further expansion of the demand for cocoa products both in the form of new cocoa-based products (such as chocolate chip cookies, chocolate sodas, etc.) and increased utilization of cocoa butter in other industries (such as the cosmetic industry); these developments will intensify in the event that cocoa bean prices remain low.

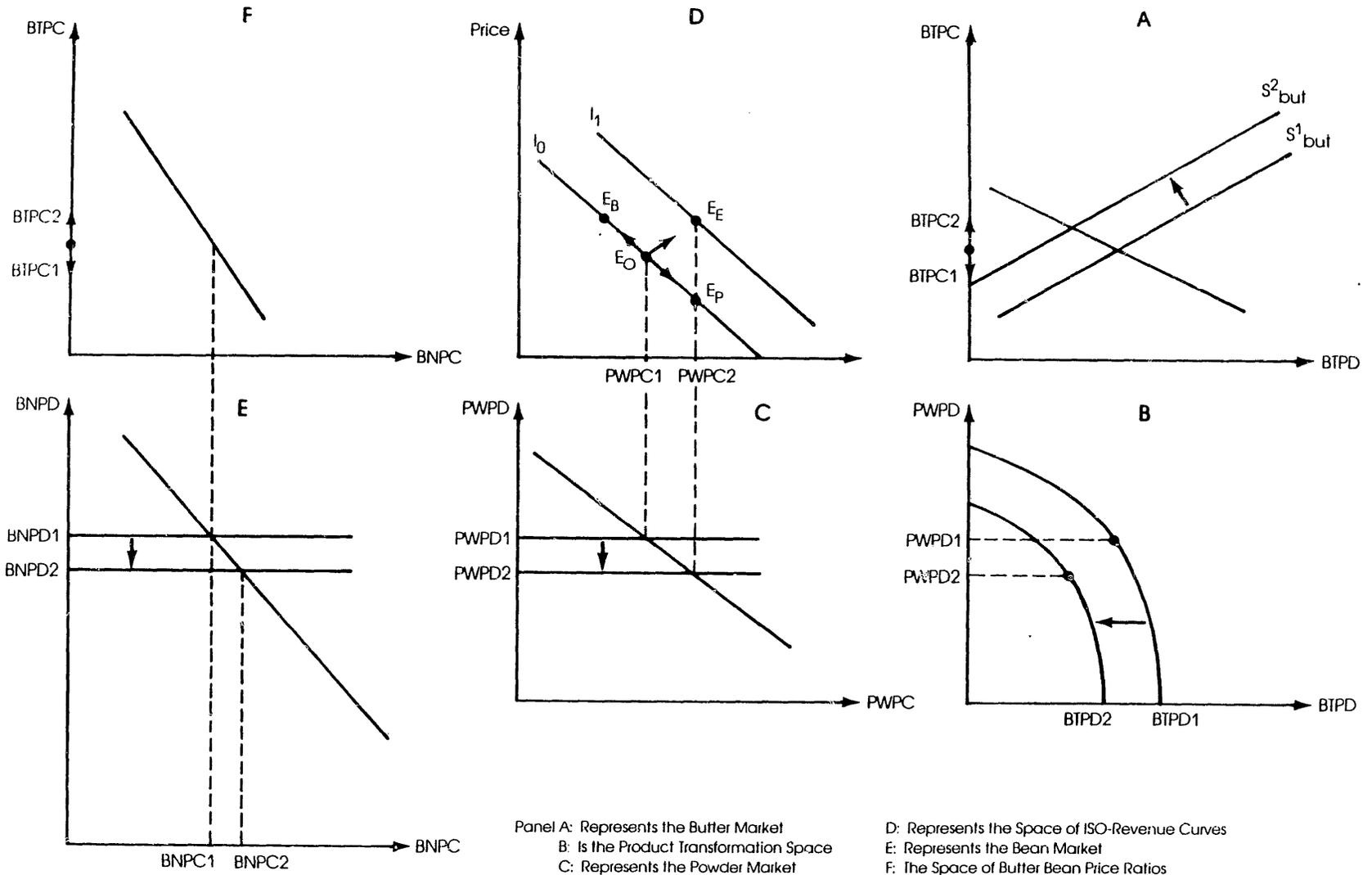
139. The study inspires two directions for further research. One is to solve the maximization problem of the cocoa bean processor using the economic and technological facts brought forth in the study. The second is to improve the quality of data on cocoa products.

Cocoa Bean and Product Market Dynamics Given a Supply Shock

A shock in the supply of beans is transmitted to the product markets. However, because cocoa butter and cocoa powder are jointly produced, the equilibrium price ratio is somewhat undetermined. Manufacturers of both products have a range of equally profitable price options in a range consistent with the product transformation space.

In the case illustrated by the following set of graphs, the cocoa bean market (panel E) suffers an exogenous supply decrease from BNPDI to BNPD2 in the immediate run. This decrease is passed onto the cocoa butter and cocoa powder markets (panels A and C) via the production possibility space (panel B). In panel B, a reduction in the supply of beans is represented by an inward shift of the production possibility curve resulting in a reduction of the total achievable output of cocoa butter from BTPDI to BTPD2 and of cocoa powder from PWPD1 to PWPD2. In the iso-revenue space (panel D), either Eb or Ep along the isorevenue curve I₀ can represent the manufacturers' equilibrium situation. At Eb they accept low bids on powder whereas at Ep they accept low bids on butter. In the extreme case where market conditions induce a simultaneous increase in butter and powder prices, the equilibrium is at Ee on another isorevenue curve I₁. Thus, from the original situation E₀, new outcomes could be on the same price curve in either direction depending on how the product arbitrage is made or on the price expansion curve in the direction of Ee.

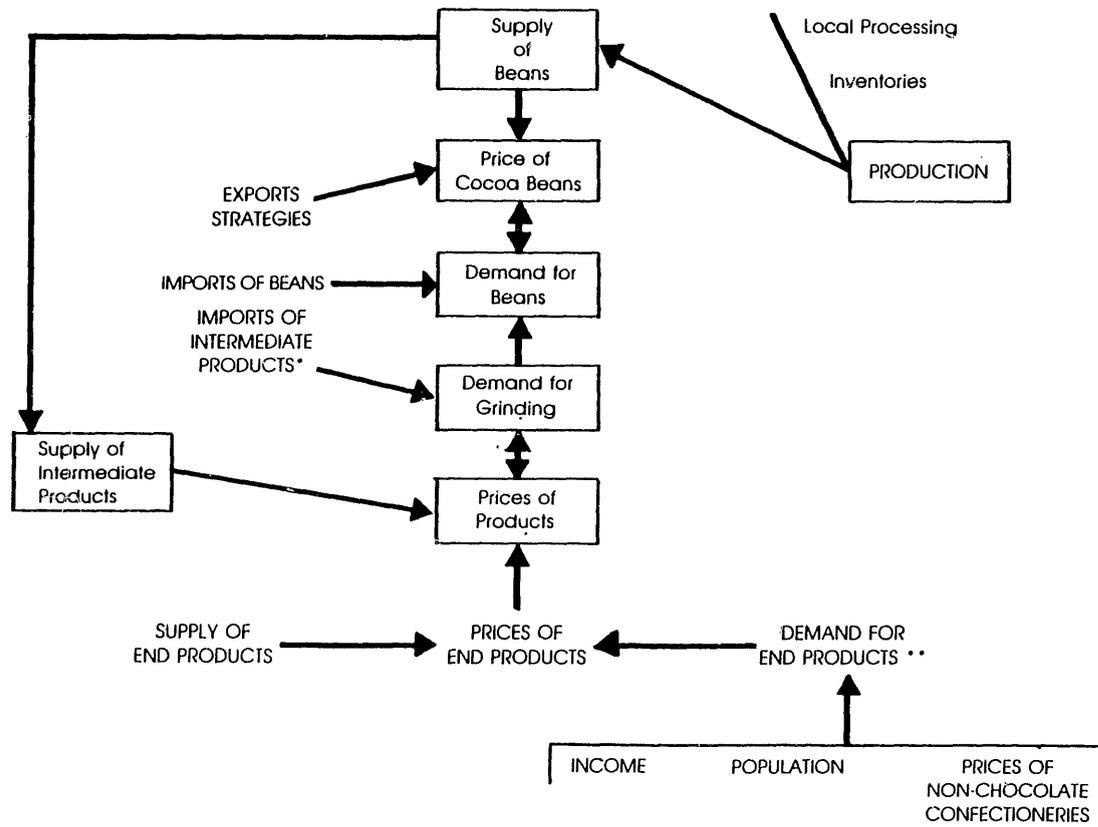
Annex A: Cocoa Market Price Dynamics Under a Bean Supply Shortage Shock



Panel A: Represents the Butter Market
 B: Is the Product Transformation Space
 C: Represents the Powder Market
 D: Represents the Space of ISO-Revenue Curves
 E: Represents the Bean Market
 F: The Space of Butter Bean Price Ratios

ANNEX B

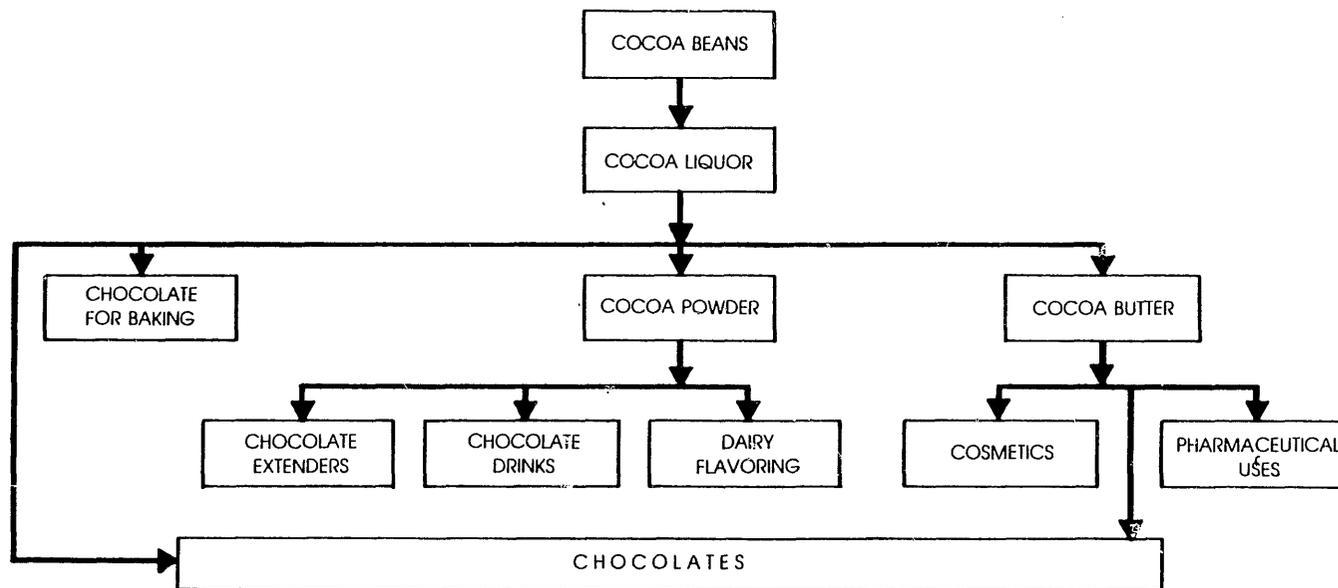
COCOA MARKET RELATIONSHIPS



*Liquor, butter and powder

**Solid plain chocolate, solid chocolate with other food ingredients, chocolate coated goods etc.

COCOA PRODUCT FLOW CHART.



SOURCE: EPDCS.

World Bank—30999:3

COCOA PRODUCT OUTPUT USAGE & PRICE DETERMINATION

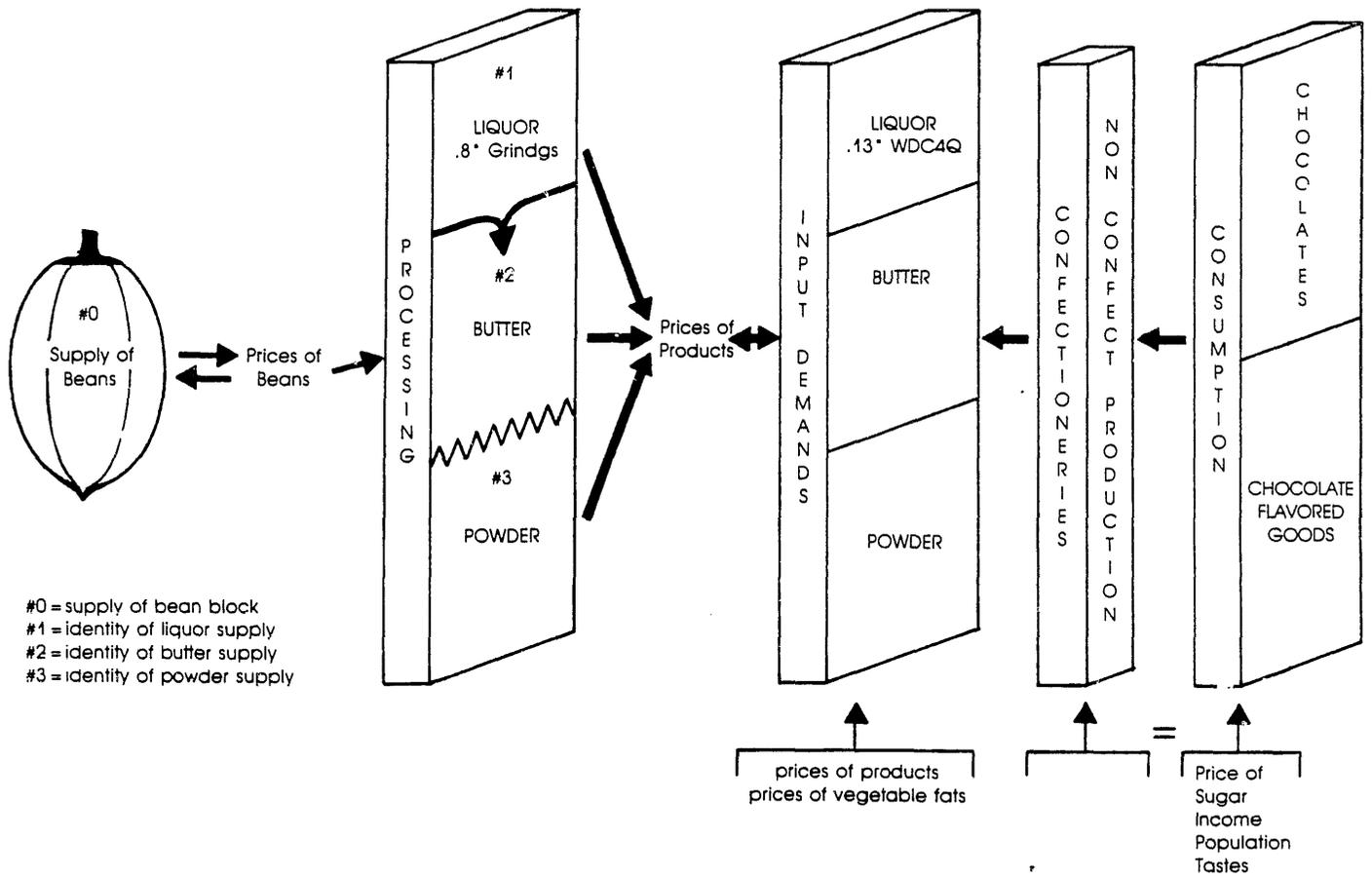


TABLE E.1: ESTIMATION RESULTS FOR MAIN EQUATIONS OF THE COCOA PRODUCTS SUBMODEL

ANNEX E

	CHOCOLATE PRODUCTION	BEAN PRICE	GRINDINGS	LIQUOR PRICE	BUTTER PRICE	POWDER PRICE	COCOA BUTTER DEMAND	COCOA POWDER DEMAND	CHOCOLATE PRICE
Constant	4.60 (10.12)	16.21 (4.40)	0.26 (1.35)	2,262.18 (2.97)	-2,874.40 (-1.44)	1,190.35 (2.53)	140.53 (4.39)	191.01 (4.65)	
FBTI	25.17 (5.23)					9.60 (1.23)			
BTFC ₋₁	-0.12 (2.69)								
CHFC ₋₁	0.51 (4.12)								
BNGRDM			0.88 (16.52)						
FTMG			0.03 (2.20)						
BNGR ₋₁			0.10 (1.79)						
BNPCD			-0.02 (-3.89)	0.80 (9.09)					
LQPC ₋₁				-0.91 (-12.60)					
STGRR		-0.86 (-3.64)		-1309.10 (-2.00)					
CHPD				0.64 (3.06)			0.02 (1.78)		
BNGR				-2.39 (-4.10)					-1.11 (-1.73)
STGRRBTFC					-1.65 (-5.60)				
BTLOUSR					1,542.90 (1.66)		7.77		
STGRRFWPC					-1.66 (-2.87)				
STGRRLQPC					5.21 (5.85)				
LQPC						1.33 (15.70)			

TABLE E.1: ESTIMATION RESULTS FOR MAIN EQUATIONS OF THE COCOA PRODUCTS SUBMODEL (continued)

	CHOCOLATE PRODUCTION	BEAN PRICE	GRINDINGS	LIQUOR PRICE	BUTTER PRICE	POWDER PRICE	COCOA BUTTER DEMAND	COCOA POWDER DEMAND	CHOCOLATE PRICE
BTPC						-0.76 (-7.52)			0.26 (4.69)
PWPC						-0.68 (-11.64)			
UNUSFT						-27.43 (-2.06)			
BSCHPDR								28.65 (2.01)	
PWDM ₋₁								0.44 (3.46)	
BTDM ₋₁							0.75 (7.77)		
PWCHPCR								-40.27 (-4.28)	
BTBNPCR							-22.62 (-1.19)		
BTFTPCR							-0.62 (-1.07)		
BNCHPCR							-41.96 (-2.60)		
BNDM		1.51 (3.34)							1.72 (2.78)
BNTS		-2.95 (5.86)							
MUV ₋₁		1.22 (8.74)							
XRTINDEX		-0.94 (-1.44)							
DW STATISTIC	1.88	1.82	2.02	1.70	1.80	1.98	2.40	1.60	1.50
R ²	0.95	0.95	0.99	0.93	0.80	0.94	0.89	0.75	0.95

Note: All variable names are defined in Table 5 except the following:

- BNTS = World total supply of cocoa bean.
- XRTINDEX = Consumption weighted index of exchange rate for the major cocoa importing countries.
- STGRRBTPC = Stocks-to-grindings ratio times cocoa butter price.
- STGRRPWPC = Stocks-to-grindings ratio times cocoa powder price.
- STGRRLQPC = Stocks-to-grindings ratio times cocoa liquor price.

SOURCE: INTERNATIONAL COMMODITY MARKETS DIVISION, INTERNATIONAL ECONOMICS DEPARTMENT.

Cocoa Bean and Product Price Elasticity Relationships 1/

- Let
- X = Quantity of beans
 - X₁ = Quantity of cocoa butter
 - X₂ = Quantity of cocoa powder
 - W₁ = yield of cocoa butter per unit of cocoa bean
 - W₂ = yield of cocoa powder per unit of cocoa bean

Then $X_1 = W_1X$ and $X_2 = W_2X$ (1)

If P₁ and P₂ are the prices of butter and powder respectively, then the average revenue per unit of cocoa bean obtained from the sale of cocoa butter and cocoa powder is

$$A_x = W_1P_1 + W_2P_2 \quad (2)$$

Let us ignore marketing and processing costs.

Implicit partial differentiation of (2) with respect to A_x gives

$$1 = W_1 \left(\frac{\delta P_1}{\delta X} \right) \left(\frac{\delta X}{\delta A_x} \right) + W_2 \left(\frac{\delta P_2}{\delta X} \right) \left(\frac{\delta X}{\delta A_x} \right) \quad (3)$$

$$1 = \frac{\delta X}{\delta A_x} \left[W_1 \left(\frac{\delta P_1}{\delta X} \right) + W_2 \left(\frac{\delta P_2}{\delta X} \right) \right] \frac{\partial x}{\partial A_x} \quad (4)$$

Thus

$$\frac{\delta X}{\delta A_x} = \left[W_1 \left(\frac{\delta P_1}{\delta X} \right) + W_2 \left(\frac{\delta P_2}{\delta X} \right) \right]^{-1} \quad (5)$$

1/ This annex draws heavily from Houck (1964).

Let the price elasticity of the demand for beans be denoted as

$$E_x = \frac{\delta X}{\delta A_x} \times \frac{A_x}{X} \quad (6)$$

Multiplying both sides of (5) by $\frac{A_x}{X}$ yields

$$E_x = [W_1 X \left(\frac{\delta P_1}{\delta X}\right) + W_2 X \left(\frac{\delta P_2}{\delta X}\right)]^{-1} A_x \quad (7)$$

Using (1) we can write

$$X = \frac{X_i}{W_i} \quad i = 1, 2 \quad X = W_i \quad (8)$$

Implicit partial differentiation of (8) gives

$$1 = \frac{1}{W_i} \left(\frac{\partial X_i}{\partial P_i}\right) \left(\frac{\partial P_i}{\partial X}\right) \quad i = 1, 2 \quad (9)$$

Multiplying both sides by $\frac{P_i}{X_i}$

$$\frac{P_i}{X_i} = \frac{E_i}{W_i} \left(\frac{\partial P_i}{\partial X}\right) \quad i = 1, 2 \quad (10)$$

where E_i is the price elasticity of butter or powder for $i = 1$ or $i = 2$

Further transformation gives:

$$\frac{\partial P_i}{\partial X} = \left(\frac{W_i}{X_i}\right) \left(\frac{P_i}{E_i}\right) \quad i = 1, 2 \quad (11)$$

Using (11) to substitute into (7) yields

$$E_x = (W_1 P_1 \frac{1}{E_1} + W_2 P_2 \frac{1}{E_2})^{-1} \cdot A_x \quad (12)$$

which is finally

$$E_x = \frac{W_1 P_1 + W_2 P_2}{\frac{1}{E_1} (W_1 P_1) + \frac{1}{E_2} W_2 P_2} \quad (13)$$

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